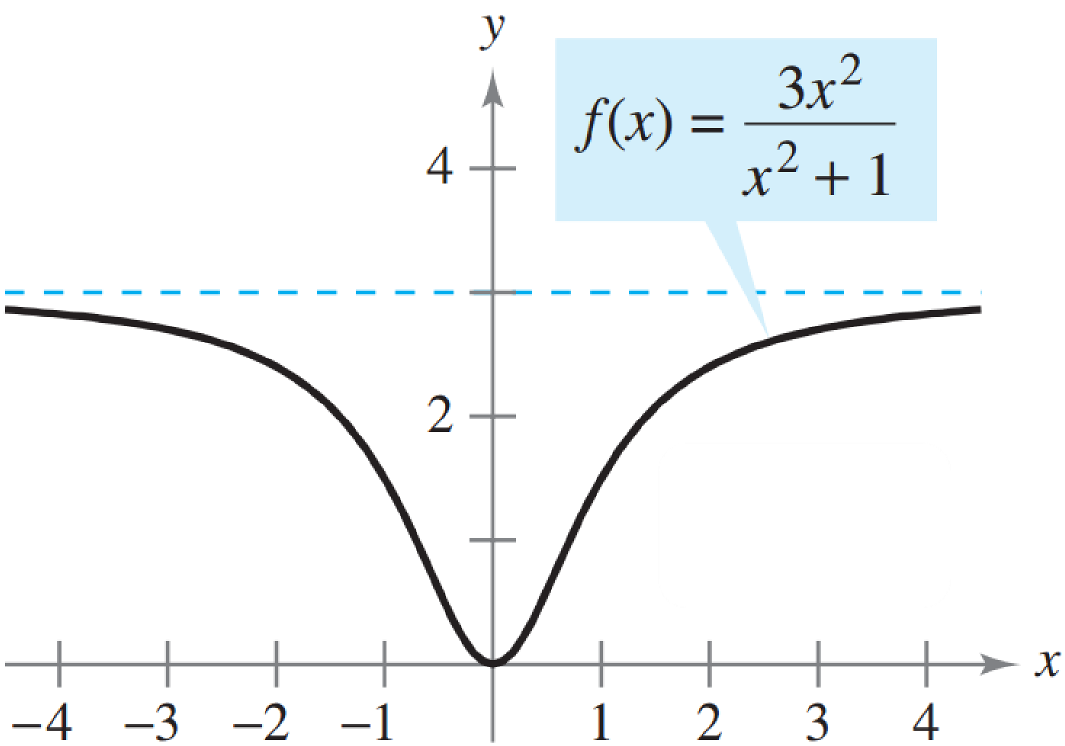
Limits at Infinity

Horizontal Asymptotes

How do we determine the limit as $x \to \infty$ or $x \to -\infty$?

Using a graph and table

$$f(x) = \frac{3x^2}{x^2 + 1}$$



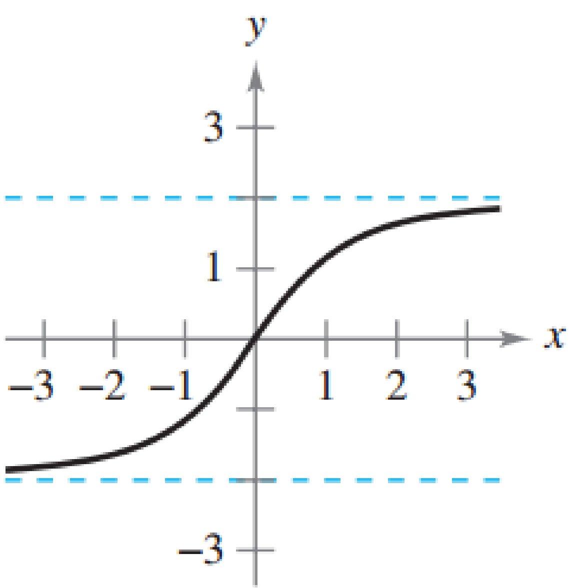
$$\lim_{x \to -\infty} f(x) = 3 \qquad \lim_{x \to \infty} f(x) = 3$$

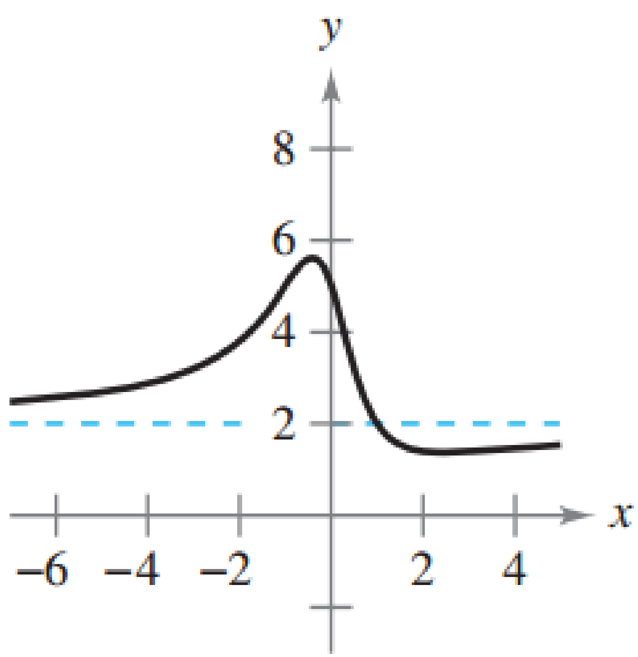
x	$-\infty \leftarrow$	-100	-10	-1	0	1	10	100	$\rightarrow \infty$
$f(x)$	$3 \leftarrow$	2.9997	2.97	1.5	0	1.5	2.97	2.9997	$\rightarrow 3$

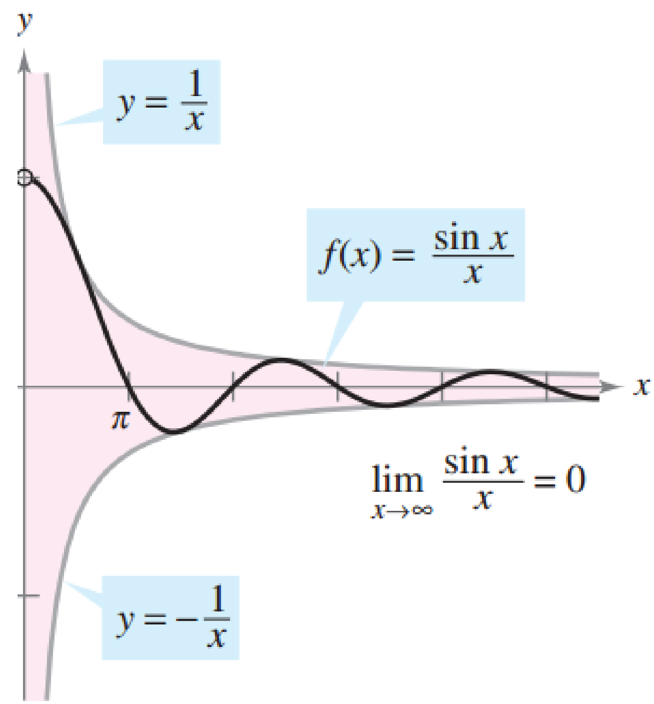
Horizontal Asymptote

The line $y = L$ is a horizontal asymptote of f if:

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad \lim_{x \to \infty} f(x) = L$$







Trigonometric Limits and Squeeze Theorem

a. $\lim_{x \to \infty} \sin x$ **b.** $\lim_{x \to \infty} \frac{\sin x}{x}$

a. Since this limits oscillates between -1 and 1, the limit DNE.

b. For this function we will use the squeeze theorem

Squeeze Theorem

- One way of finding a limit is by "squeezing" a function between two other functions.

$$-1 \leq \sin x \leq 1$$

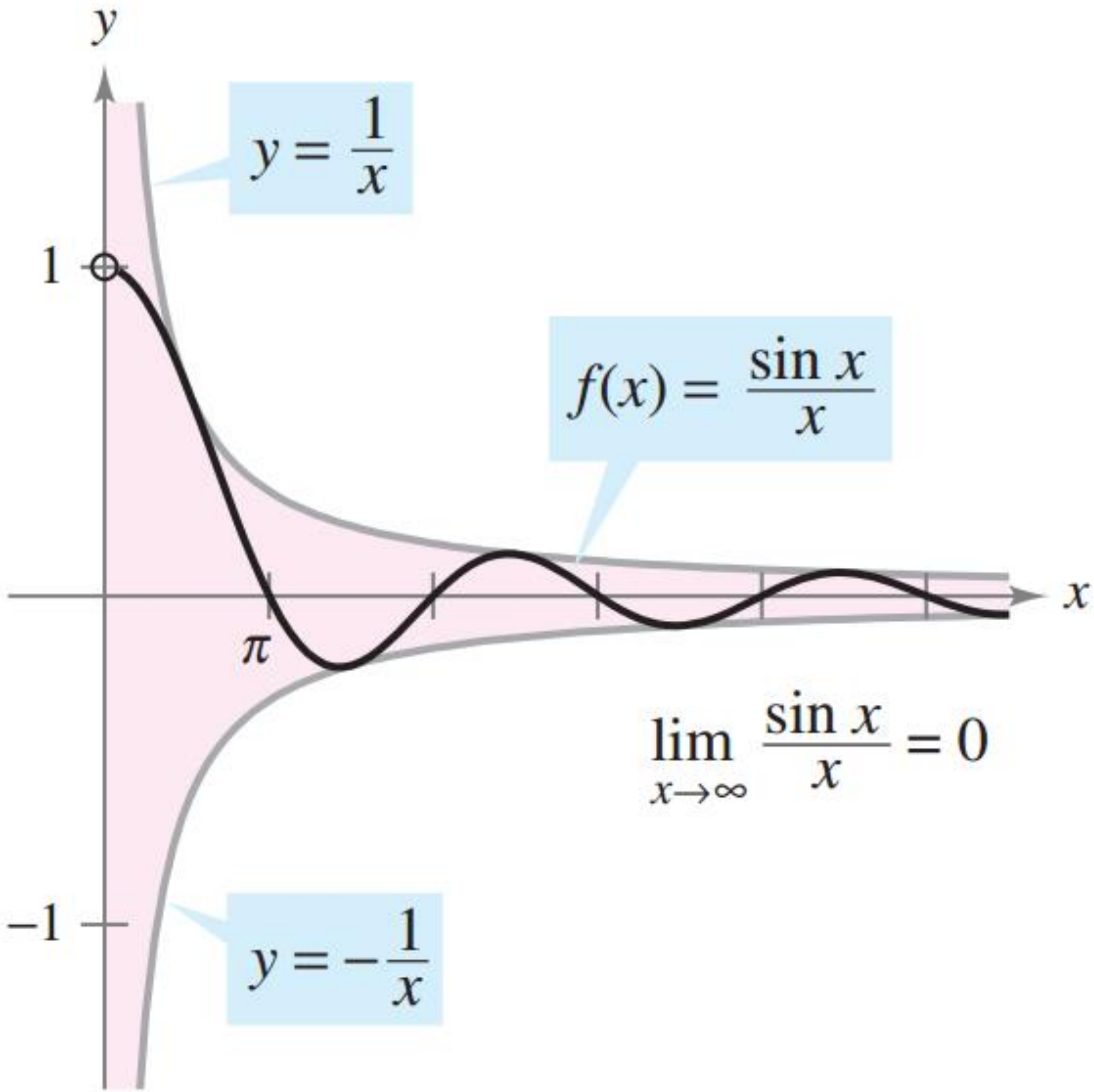
$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$\lim_{x \to \infty} (-1/x) = 0$$

$$\lim_{x \to \infty} (1/x) = 0$$

$$\lim_{x \to \infty} \frac{\sin x}{x} = 0$$

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$



Horizontal Asymptotes

*A function can have, at most, 2 horizontal asymptotes, one to the right and one to the left.

*When we have a horizontal asymptote at $y = L$ we say that the limit exists and is equal to L.

Also remember that $\frac{c}{\infty}$ and $\frac{c}{-\infty}$ both "equal" 0.

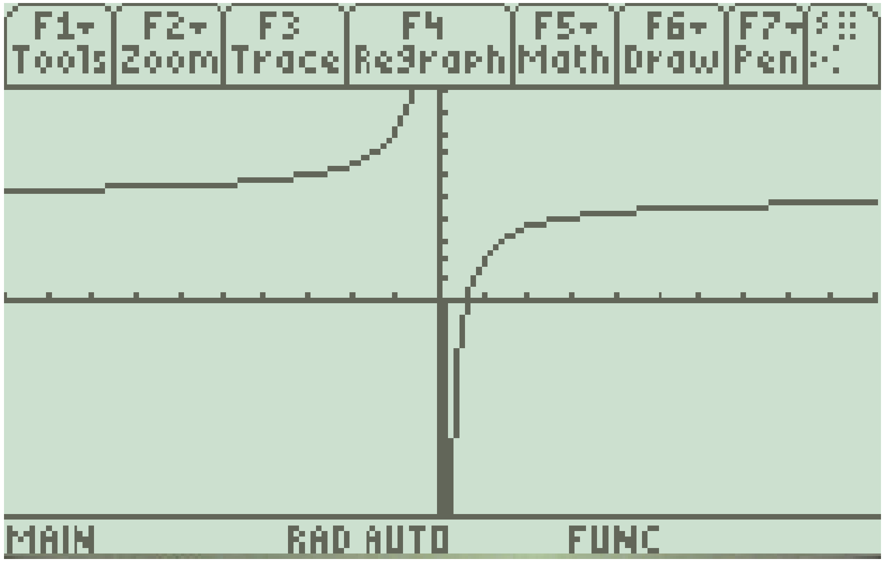
$$\lim_{x\to\infty}\left(5-\frac{3}{x}\right)$$

$$\lim_{x\to-\infty}\left(5-\frac{3}{x}\right)$$

$$\lim_{x\to 0}\left(5-\frac{3}{x}\right)$$

$$\lim_{x\to 0^+}\left(5-\frac{3}{x}\right)$$

$$\lim_{x\to 0^-}\left(5-\frac{3}{x}\right)$$



If you have a rational function then one option is to divide by the x with the highest degree in denominator.

$$\lim_{x \to \infty} \frac{2x + 5}{3x^2 + 1}$$

$$= \lim_{x \to \infty} \frac{(2/x) + (5/x^2)}{3 + (1/x^2)}$$

$$= \frac{0}{3} = 0$$

$$\lim_{x\to\infty} \frac{2x^2 + 5}{3x^2 + 1}$$

$$= \lim_{x\to\infty} \frac{2 + (5/x^2)}{3 + (1/x^2)}$$

$$= \frac{2}{3}$$

$$\lim_{x\to\infty} \frac{2x^3 + 5}{3x^2 + 1}$$

$$= \lim_{x\to\infty} \frac{2x + (5/x^2)}{3 + (1/x^2)}$$

$$= \frac{\infty}{3} = \infty$$

$$\lim_{x\to\infty} \frac{2x + 5}{3x^2 + 1} = 0$$

$$\lim_{x\to\infty} \frac{2x^2 + 5}{3x^2 + 1} = \frac{2}{3}$$

$$\lim_{x\to\infty} \frac{2x^3 + 5}{3x^2 + 1} = \infty$$

Finding limits to $\pm\infty$ for Rational Functions

1. If highest degree of numerator is *less than* highest degree of denominator, then

   limit = 0

2. If highest degree of numerator is *equal to* highest degree of the denominator, then

   limit = ratio of the lead coefficients

3. If highest degree of numerator *greater than* highest degree of the denominator then

   limit = $\pm\infty$

$$\lim_{x\to\infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$$

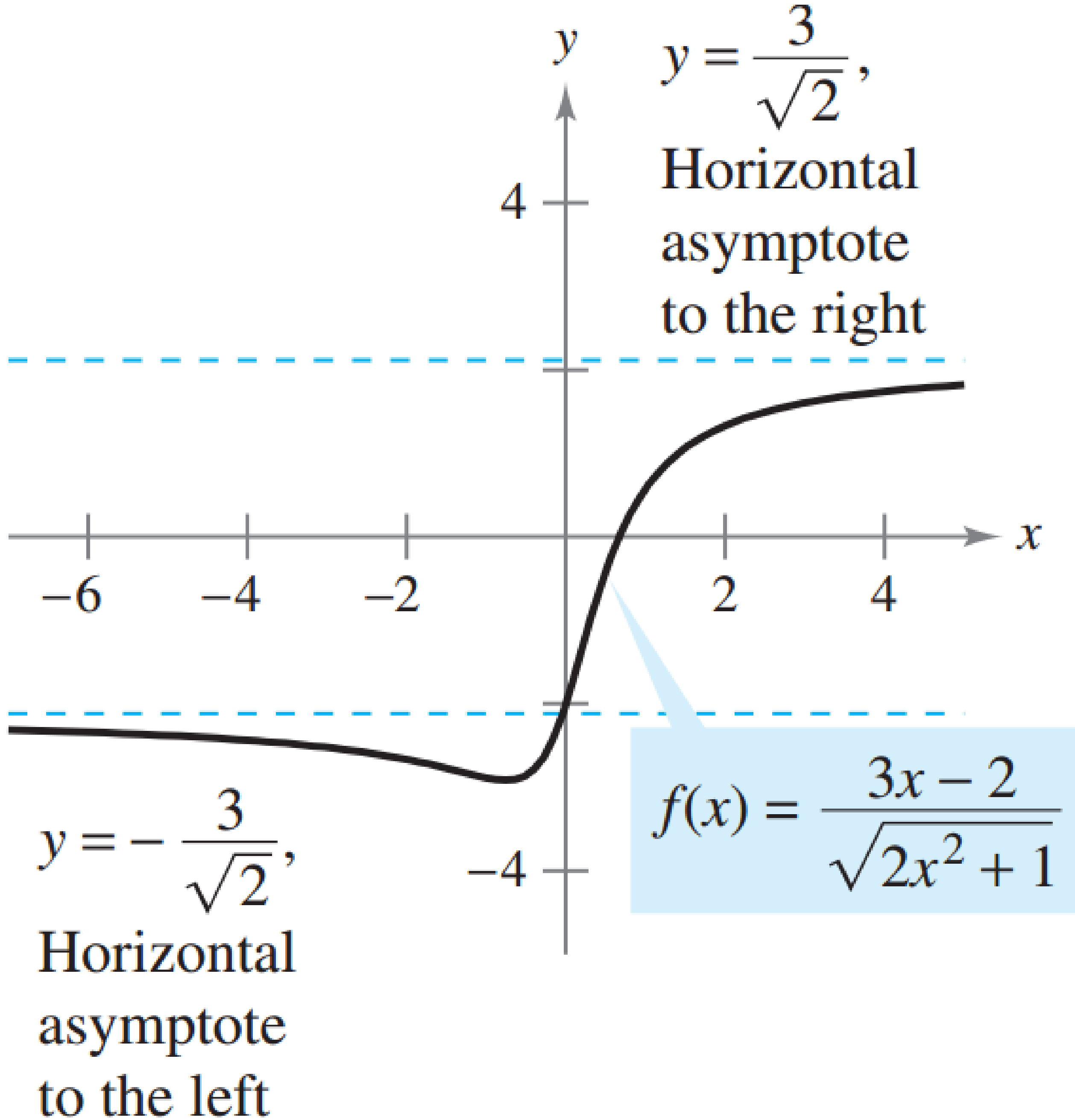
$$= \frac{\dfrac{3x - 2}{x}}{\dfrac{\sqrt{2x^2 + 1}}{\sqrt{x^2}}} = \frac{3 - \dfrac{2}{x}}{\sqrt{2 + \dfrac{1}{x^2}}}$$

$$\lim_{x\to\infty} \frac{3 - \dfrac{2}{x}}{\sqrt{2 + \dfrac{1}{x^2}}} = \frac{3}{\sqrt{2}}$$

$$\lim_{x \to -\infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$$

$$= \frac{\dfrac{3x - 2}{x}}{\dfrac{\sqrt{2x^2 + 1}}{-\sqrt{x^2}}} = \frac{3 - \dfrac{2}{x}}{-\sqrt{2 + \dfrac{1}{x^2}}}$$

$$\lim_{x \to -\infty} \frac{3 - \dfrac{2}{x}}{-\sqrt{2 + \dfrac{1}{x^2}}} = -\frac{3}{\sqrt{2}}$$



y
$y = \frac{3}{\sqrt{2}}$,
Horizontal
asymptote
to the right
4
x
−6
−4
−2
2
4
$f(x) = \frac{3x - 2}{\sqrt{2x^2 + 1}}$
$y = -\frac{3}{\sqrt{2}}$,
−4
Horizontal
asymptote
to the left

Homework

Limits at Infinity

Section 3.5

(1-8, 9-33 odd)

Graph sinx and cos x

What does your graph look like?
Where are vertical asymptotes?
What happens as you approach vertical asymptotes from the left and right?