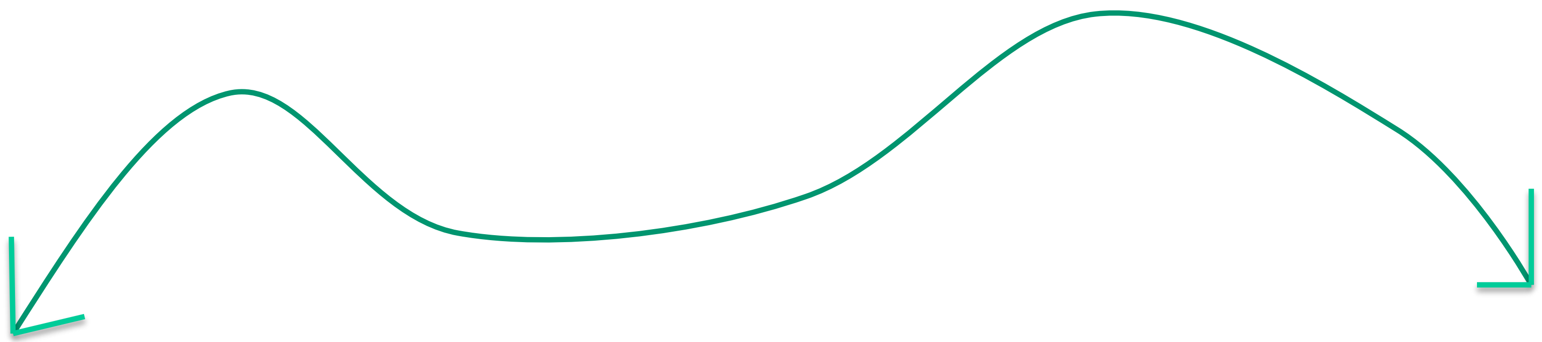
Continuity and One Sided Limits

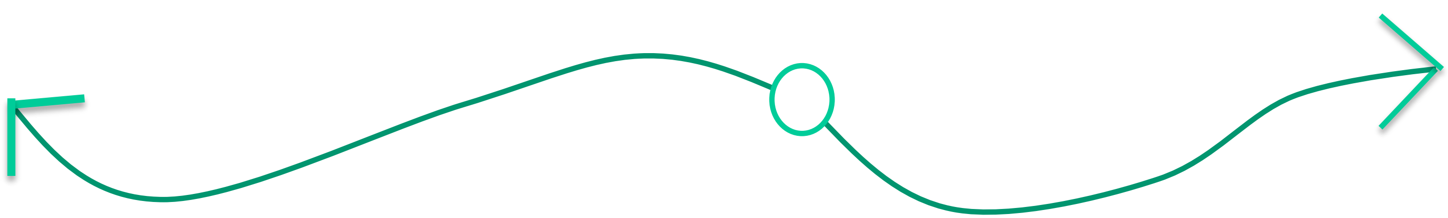
Continuity is very simple.

Continuity comes from the word continuous.

If you can draw a graph without picking up your pencil, then the function is continuous.



If the function is not continuous, we say it is

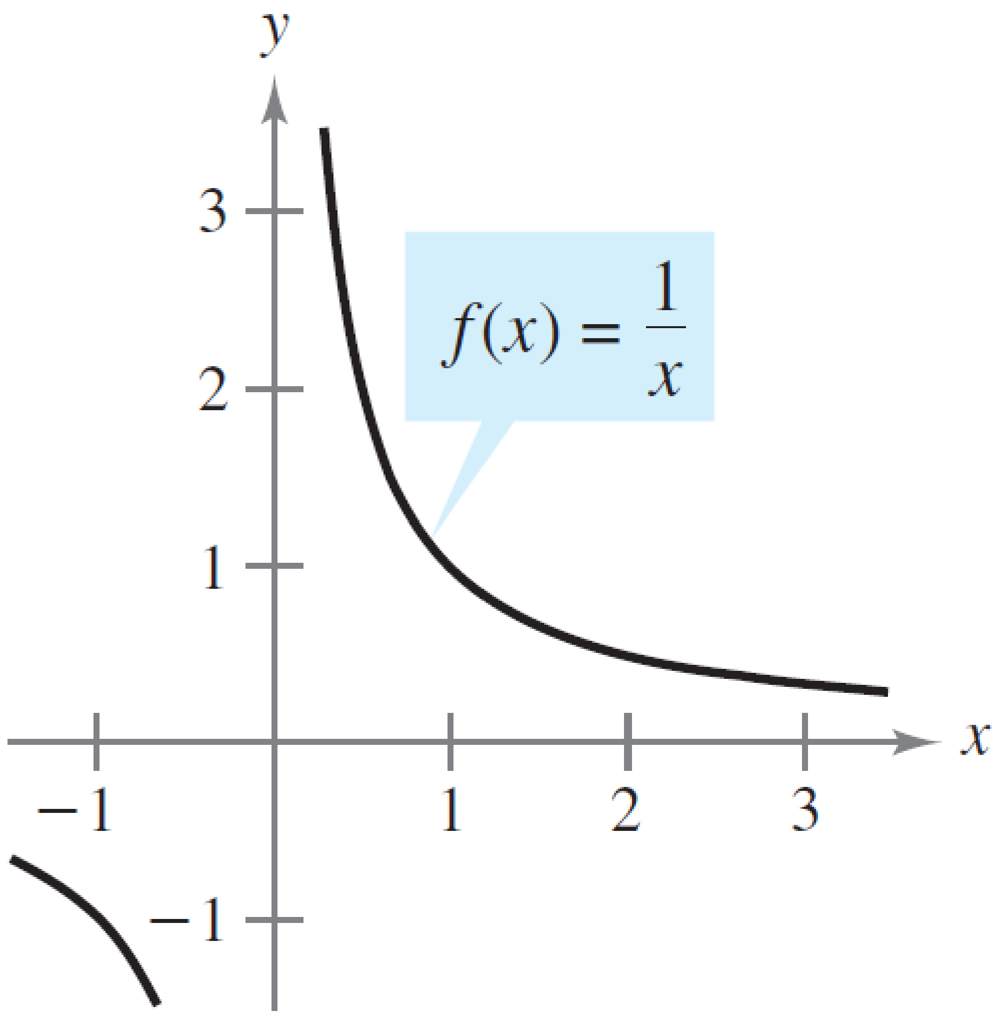


Mathematical Definition of Continuity

- A function is continuous at c, if the following are all true.
  - »1.
  - »2.
  - »3.

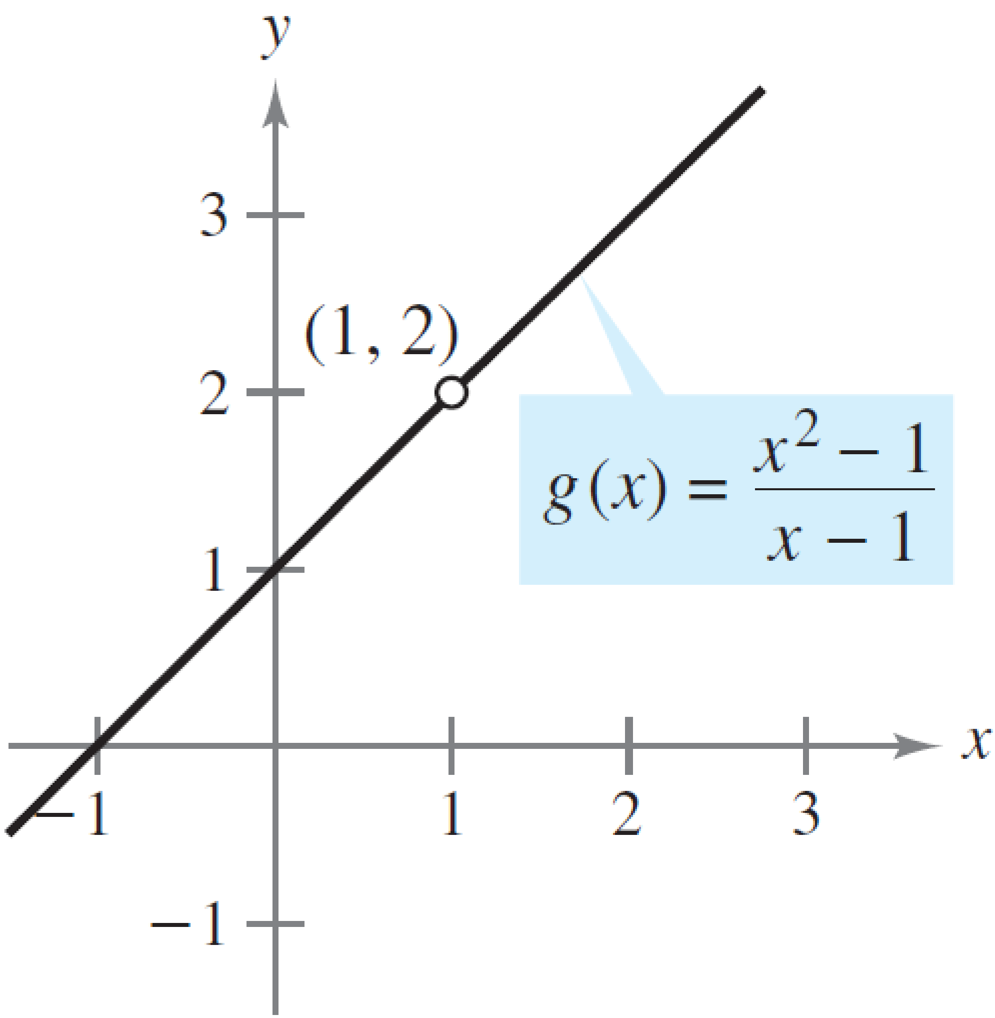
Is it always continuous?

$$f(x) = \frac{1}{x}$$



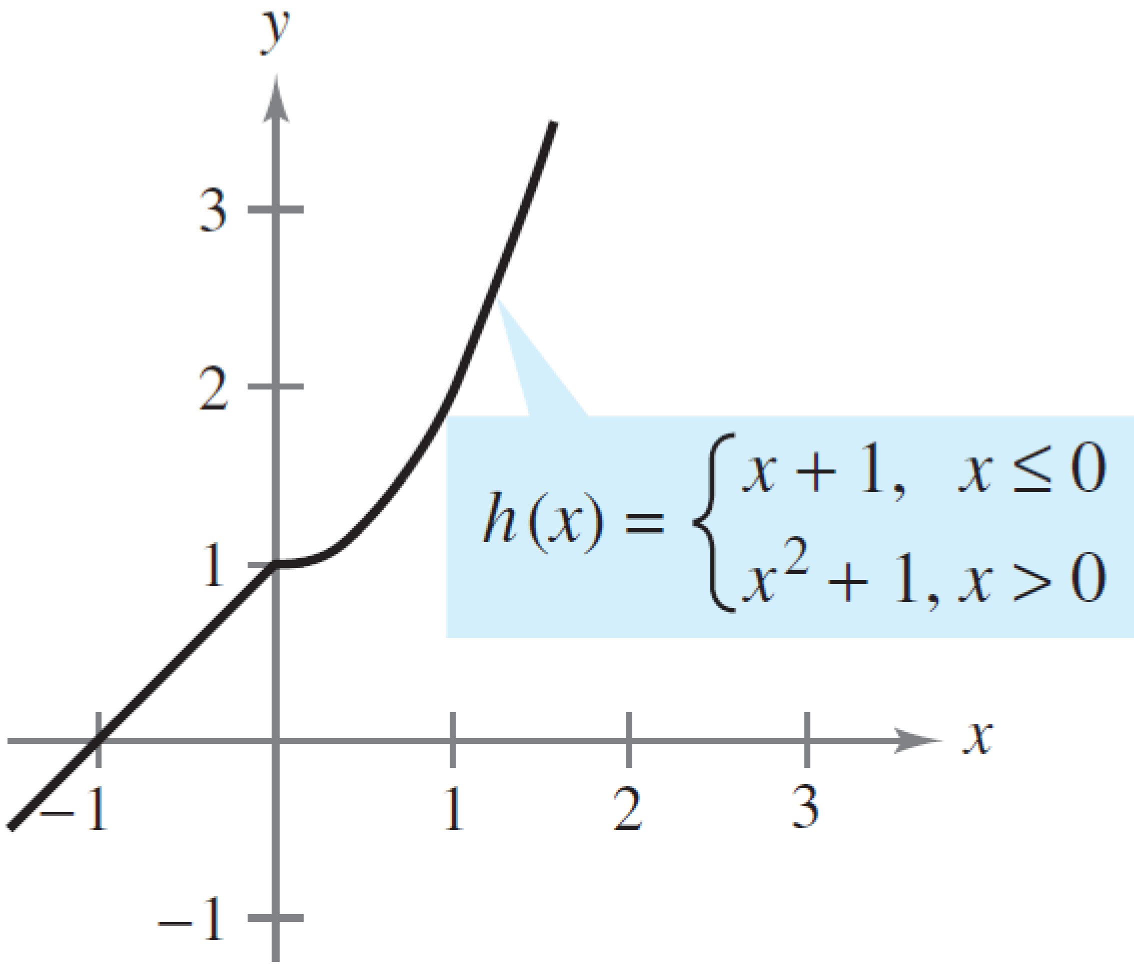
Is it always continuous?

$$f(x) = \frac{x^2 - 1}{x - 1}$$



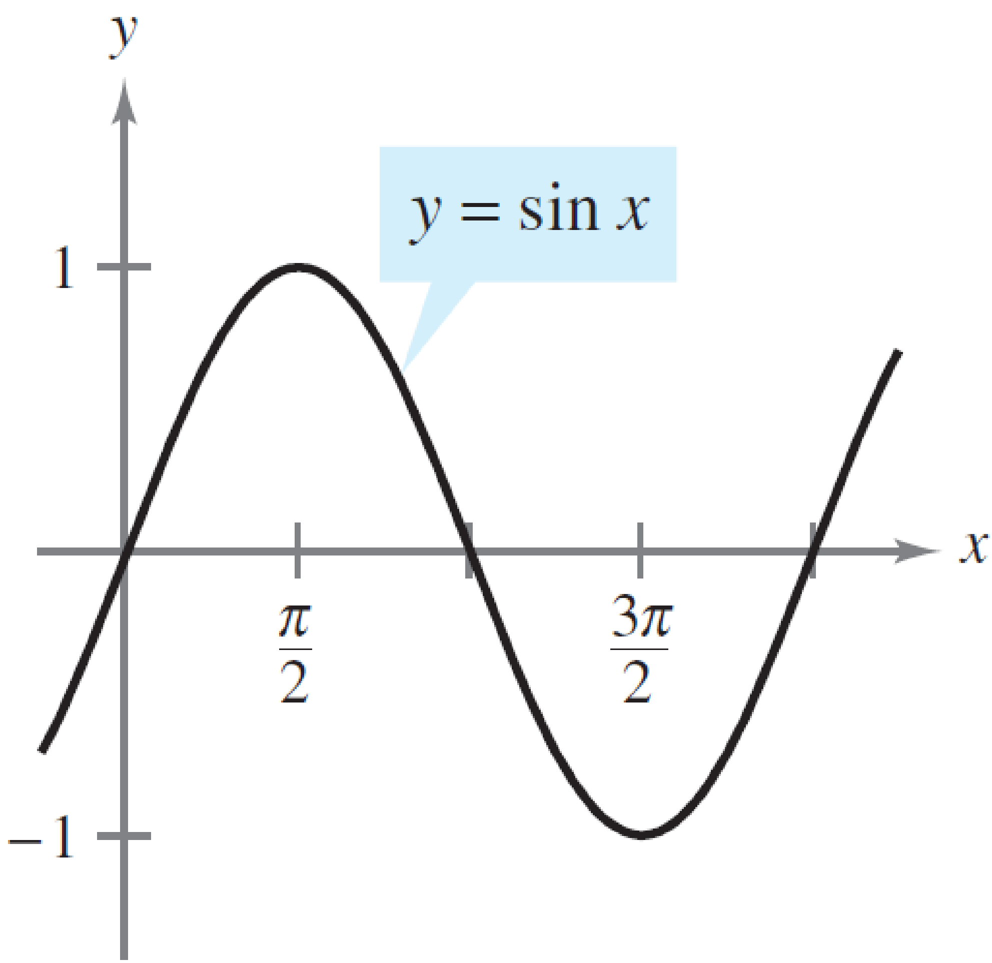
Is it always continuous?

$$f(x) = \begin{cases} x+1, x \leq 0 \\ x^2+1, x > 0 \end{cases}$$

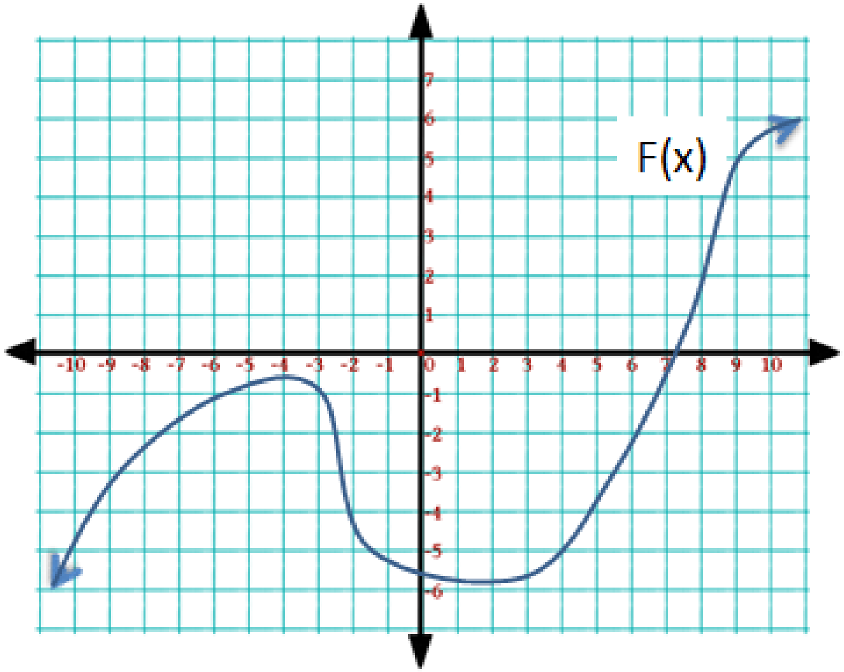


Is it always continuous?

$$f(x) = \sin x$$



If a function is continuous at a point, what do we know about its limit at that point?



Discontinuity

- If a function is not continuous, we say that it is discontinuous.

Four Types of Discontinuity.

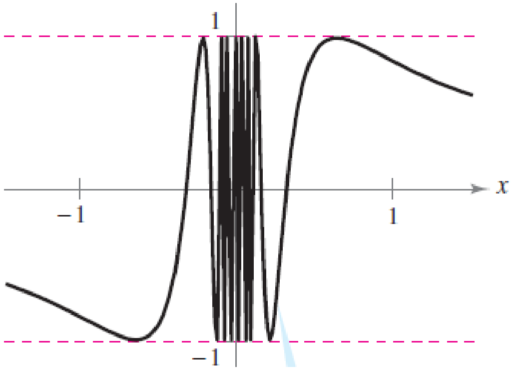
1. Point of Discontinuity.

2. Infinite Discontinuity.

3. Jump Discontinuity.

4. Oscillating Function.

$$y = \sin\frac{1}{x}$$



Removable vs Non-removable Discontinuities

1. **Removable Discontinuity.**

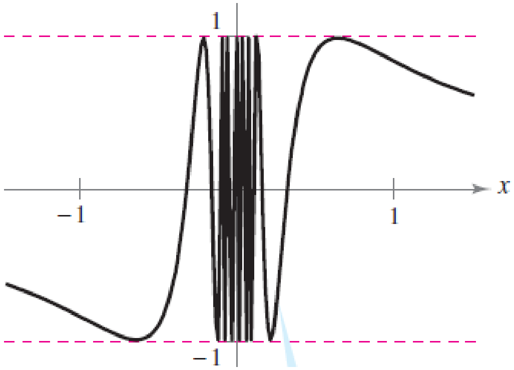
Non-removable Discontinuities

2. **Infinite Discontinuity.**

3. **Jump Discontinuity.**

4. **Oscillating Function.**

$$y = \sin\frac{1}{x}$$



One-Sided Limits

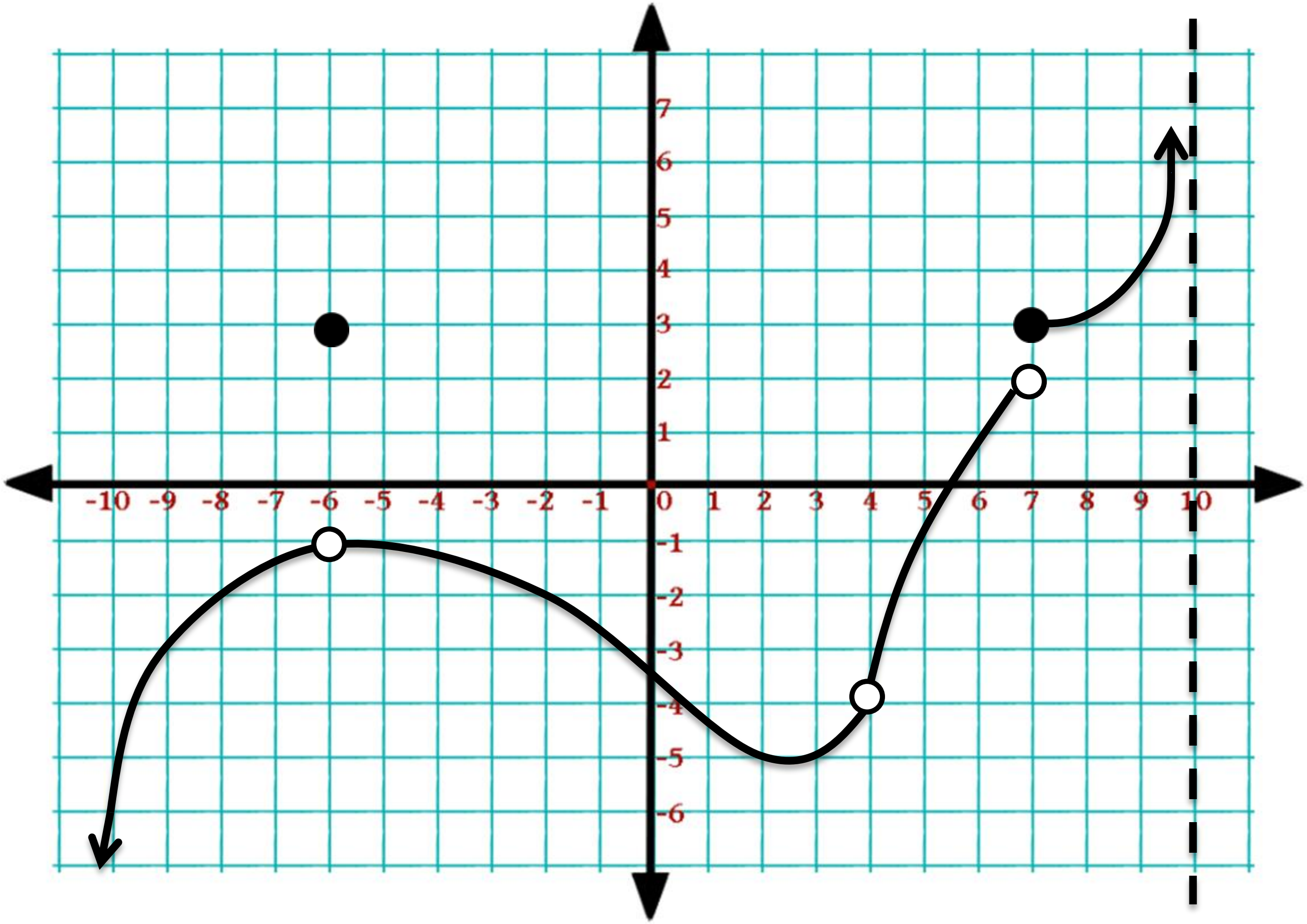
Sometimes, we will be interested in finding a limit only from one side.

To find the limit from the right, you will use a $+$.

$$\lim_{x \to c^+} f(x) = L$$

To find the limit from the left, you will use a $-$.

$$\lim_{x \to c^-} f(x) = L$$



$$\lim_{x \to 7^-} f(x) =$$

$$\lim_{x \to 7^+} f(x) =$$

$$\lim_{x \to -6^-} f(x) =$$

$$\lim_{x \to -6^+} f(x) =$$

$$\lim_{x \to 10^-} f(x) =$$

One Sided Limits Analytically

- When evaluating a one sided limit, just plug in the number as if it was not one sided.

$$\lim_{x \to 1^+} \frac{x^2 + x + 2}{x + 1} = 2$$

- Generally the limit will be the same unless there is an Asymptote $\left(\frac{\#}{0}\right)$ or there is a jump discontinuity (usually in a piecewise).

$$\lim_{x \to 5} \left(\frac{x}{x-5}\right) \qquad \lim_{x \to 5^+} \left(\frac{x}{x-5}\right) \qquad \lim_{x \to 5^-} \left(\frac{x}{x-5}\right)$$

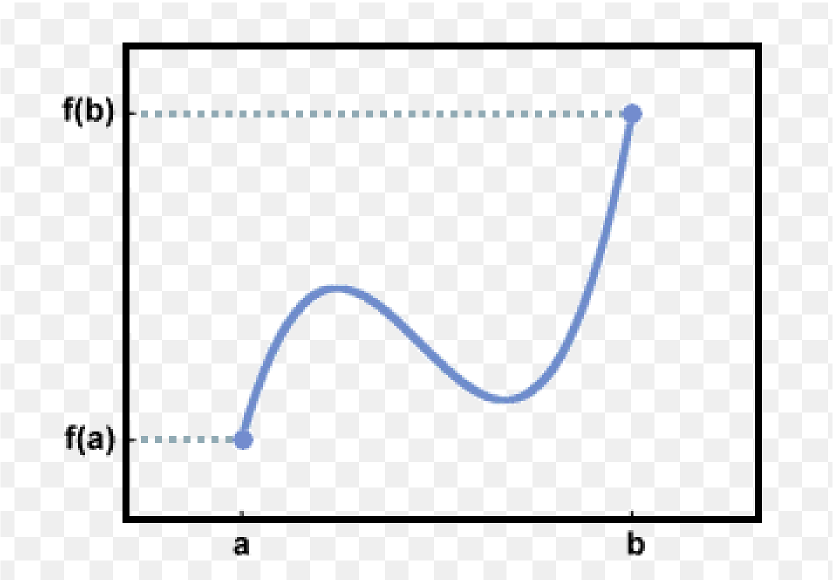
$$\lim_{x \to 5^+} \left(\frac{x}{x - 5} \right)$$

$$\lim_{x \to 5^-} \left(\frac{x}{x - 5} \right)$$

Intermediate Value Theorem

- The IVT is an existence theorem, it guarantees the existence of something.

- It states that if a function is continuous on $[a, b]$ and k is a number between $f(a)$ and $f(b)$, there exists a c between a and b such that $f(c) = k$.



Free Response IVT problem (2007 #3)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

Solution:

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.


"That's all Folks!"

Homework Day 1

Section 1.4
P. 78 (1-11 odd, 17-20, 36-40, 43, 46, 47)
P. 78 (75-85 odd) (All IVT Problems)(Optional)

-Removable Discontinuity = Point of Discontinuity

-Should be able to do all without calculator!
Can check with calculator though.