Integrate the following function.....

$$\int \frac{1}{x-3} - \frac{1}{x-2}\, dx$$

$$= \ln|x - 3| - \ln|x - 2| + C$$

Now what if we disguise this equation a bit by combining……

$$\int \frac{1}{x-3} - \frac{1}{x-2}\,dx = \int \frac{1(x-2) - 1(x-3)}{(x-3)(x-2)}\,dx$$

$$\int \frac{x-2\ -x+3}{x^2-5x+6}\,dx = \int \frac{1}{x^2-5x+6}\,dx$$

If we are given this equation initially: $\int \frac{1}{x^2-5x+6}\,dx$

We are going to have to do some expansion in order
to put it into a form that is easy to integrate.
This process is called **PARTIAL FRACTION DECOMPOSITION**.

$$\int \frac{1}{x^2 - 5x + 6}\, dx$$

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{x - 3} - \frac{1}{x - 2}$$

$$\int \frac{1}{x^2 - 5x + 6}\, dx = \int \left(\frac{1}{x - 3} - \frac{1}{x - 2}\right) dx$$

Partial Fraction Decomposition

Integrating rational functions.

$$\int \frac{1}{x^2 - 5x + 6}\,dx$$

Because $x^2 - 5x + 6 = (x - 3)(x - 2)$

$$\frac{1}{x^2 - 5x + 6} = \frac{A}{x - 3} + \frac{B}{x - 2}$$

$$\frac{1}{x^2 - 5x + 6} = \frac{A(x-2) + B(x-3)}{(x-3)(x-2)}$$

$$1 = A(x - 2) + B(x - 3)$$

let $x = 3$ $\quad A = 1$ $\quad$ let $x = 2$ $\quad B = -1$

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{x - 3} - \frac{1}{x - 2}$$

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{x - 3} - \frac{1}{x - 2}$$

$$\int \frac{1}{x^2 - 5x + 6}\, dx = \int \left(\frac{1}{x - 3} - \frac{1}{x - 2}\right) dx$$

$$= \ln|x - 3| - \ln|x - 2| + C$$

The method of Partial Fraction Decomposition works when you are integrating a rational function and the denominator can be factored.

Rational Function = Ratio of polynomials

You will decompose/expand the rational function so it can be easily integrated.

$$\int \frac{4}{x^2 - 1}\, dx$$

$$\frac{4}{x^2 - 1} = \frac{A}{x + 1} + \frac{B}{x - 1}$$

$$4 = A(x - 1) + B(x + 1)$$

Let $x = 1$ $\quad 2B = 4$ $\quad B = 2$ $\qquad$ Let $x = -1$ $\quad -2A = 4$ $\quad A = -2$

$$\int \left(\frac{-2}{x + 1} + \frac{2}{x - 1} \right) dx$$

$$\int \frac{1}{x^2 - 1} dx = -2 \ln |x + 1| + 2 \ln |x - 1| + C$$

$$\int \frac{7}{4x^2 - 9} dx$$

$$\frac{7}{4x^2 - 9} = \frac{A}{2x + 3} + \frac{B}{2x - 3}$$

$$7 = A(2x - 3) + B(2x + 3)$$

Let $x = \frac{3}{2}$ $\quad 7 = 6B \quad B = \frac{7}{6}$

Let $x = -\frac{3}{2}$ $\quad 7 = -6A \quad A = -\frac{7}{6}$

$$\frac{1}{4x^2 - 9} = \frac{-7/6}{2x + 3} + \frac{7/6}{2x - 3}$$

$$\int \frac{-7/6}{2x + 3} + \frac{7/6}{2x - 3} dx$$

*Integrate using U-Substitution

$$-\frac{7}{12} \ln|2x + 3| + \frac{7}{12} \ln|2x - 3| + C$$

$$\int \frac{5x - 15}{x^2 - x - 12} dx$$

$$\frac{5x - 15}{x^2 - x - 12} = \frac{A}{x - 4} + \frac{B}{x + 3}$$

$$5x - 15 = A(x + 3) + B(x - 4)$$

Let $x = 4$ $\quad 5(4) - 15 = A(7)$ $\quad$ Let $x = -3$ $\quad 5(-3) - 15 = B(-7)$

$A = \frac{5}{7}$ $\quad$ $B = \frac{30}{7}$

$$\int \frac{5x - 15}{x^2 - x - 12} dx = \int \frac{5/7}{x - 4} + \frac{30/7}{x + 3} \, dx$$

$$\mathbf{\frac{5}{7} \ln |x - 4| + \frac{30}{7} \ln |x + 3| + C}$$

$$\int \frac{3x-8}{x^2-4x-5}\,dx$$

$$\frac{3x-8}{x^2-4x-5} = \frac{A}{x-5} + \frac{B}{x+1}$$

$$3x-8 = A(x+1) + B(x-5)$$

Let $x = -1$

$$3(-1) - 8 = B(-6) \quad \therefore \quad B = \frac{11}{6}$$

Let $x = -1$

$$3(5) - 8 = A(6) \quad \therefore \quad A = \frac{7}{6}$$

$$\int \frac{3x-8}{x^2-4x-5}\,dx = \int \frac{\frac{7}{6}}{x-5} + \frac{\frac{11}{6}}{x+1}\,dx$$

$$\boxed{\frac{7}{6}\ln|x-5| + \frac{11}{6}\ln|x+1| + C}$$

Homework

Logistic Growth (Demana)

Orange (6.5), Yellow/Green (7.5)

(1-19 odd, omit 7)

In all of our examples thus far, the degree of the numerator has been less than the degree of the denominator.

If it is the case that the degree of the numerator is greater than or equal to the degree of the denominator, you must reduce using “polynomial” long division.

The next few slides will help you to review this technique…….

Long "Polynomial" Division Review

$$\frac{x^2 - 9x - 10}{x + 1}$$

$$x + 1\,\overline{\big)\,x^2 - 9x - 10}$$

$$\frac{x^2 + 9x + 14}{x + 7}$$

$$\mathbf{x + 7 \overline{\smash{\big)}\, x^2 + 9x + 14}}$$

$$\frac{3x^3 - 5x^2 + 10x - 3}{3x + 1}$$

$$\mathbf{3x + 1 \overline{\smash{\big)}\, 3x^3 - 5x^2 + 10x - 3}}$$

$$\int \frac{3x^3 - 5x^2 + 10x - 3}{3x + 1} \mathrm{dx}$$

$$\int x^2 - 2x + 4 - \frac{7}{3x + 1}\, dx$$

$$\boldsymbol{\frac{1}{3}x^3 - x^2 + 4x - \frac{7}{3}\ln|3x + 1| + C}$$

$$\frac{2x^3 - 9x^2 + 15}{2x - 5}$$

Since no $\boldsymbol{x}$ we must Put $\mathbf{0}\boldsymbol{x}$.

$$2x - 5 \,\overline{\big)\, 2x^3 - 9x^2 + 0x + 15}$$

$$\int \frac{2x^3 - 9x^2 + 15}{2x - 5}\, dx$$

$$\int \left(x^2 - 2x - 5 - \frac{10}{2x - 5} \right) dx$$

$$\frac{1}{3}x^3 - x^2 - 5x - 5\ln|2x - 5| + C$$

$$\frac{4x^4 + 3x^3 + 2x + 1}{x^2 + x + 2}$$

Since no x^2 we must Put $0x^2$

$$x^2 + x + 2 \,\overline{\big)\, 4x^4 + 3x^3 + 0x^2 + 2x + 1}$$

Homework

Logistic Growth (Demana)

Orange (6.5), Yellow/Green (7.5)

(6-22 even, 7, 21)