Integration by Substitution

How do you integrate a product of two functions?

How do you undo (integrate) the chain rule?

$$\frac{d}{dx}[A] = B \qquad \int B = \quad A$$

$$y = \frac{1}{3}(x^2+1)^3 \qquad \frac{dy}{dx} = (x^2+1)^2(2x)$$

$$\int (x^2+1)^2(2x)dx = \quad \frac{1}{3}(x^2+1)^3 + C$$

$$\int (x^2 + 1)^2 (2x) dx = ?$$

Solve by $U - Substitution$.

$$u = x^2 + 1 \qquad \frac{du}{dx} = 2x \qquad du = 2x\,dx$$

$$\int (x^2 + 1)^2 (2x) dx = \int u^2 du = \frac{1}{3} u^3 + C$$

$$\frac{1}{3} (x^2 + 1)^3 + C$$

$$\int 2\mathrm{x} \cos x^2 \, dx$$

$$u = x^2 \qquad\qquad du = 2x\,dx$$

$$\int \cos u \; du$$

$$\sin u \; + C$$

$$\sin x^2 \; + C$$

Always check your answer by differentiating!!

Solving by $U - Substitution$

If........

$$\frac{d}{dx}[f(u)] = f'(u) \cdot u'$$

Then......

$$\int f'(u) \cdot u' = f(u) + C$$

Integration of a Composite Function.

IF.......

$$\frac{d}{dx}[F(g(x))] = F'(g(x))g'(x)$$

Then......

$$\int F'(g(x))g'(x)\,dx = F(g(x)) + C$$

Pattern Recognition and U-Substitution.

- You are looking to let "u" equal something whose derivative, "du," appears as well.

$$\int (x^2+5)^3 2x\,dx$$

$$u = x^2 + 5$$

$$du = 2x\,dx$$

$$\int u^3 du$$

$$\int 3x^2 \sin x^3\,dx$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\int \sin u\,du$$

*du must appear, multiplied and in the denominator.

$$\int \sqrt[3]{(1-2x^2)}\,(-4x)dx$$

The Integral Constant Multiplier Rule

$$\int kf(x)\,dx = k\int f(x)\,dx$$

You can pull a constant (k) out of the integral.

$$\int \frac{1}{2}\cdot 2\,x(x^2+1)^2dx \qquad u = x^2+1 \qquad du = 2xdx$$

$$\frac{1}{2}\int u^2du = \frac{1}{2}\left(\frac{1}{3}u^3\right) = \frac{1}{6}u^3 + C = \boxed{\frac{1}{6}(x^2+1)^3 + C}$$

*Always check your final answer by differentiating.

$$\int x^3 \cos(x^4 + 2)\, dx$$

$$\int e^{3x}\,dx$$

$U - Substitution$ allows us to take the integral of many important functions.

$$\int \tan x \, dx$$

$$\int \frac{\sin x}{\cos x} \, dx$$

$$u = \cos x$$

$$du = -\sin x \; dx$$

$$-\int \frac{1}{u} \, du$$

$$-\int u^{-1} \, du$$

$$-\ln u + C$$

$$-\ln \cos x + C$$

*Note that du must be in the numerator or substitution does not work.

$U - Substitution\ Summary$

-We are integrating the chain rule.

-We make all “x” into “u” and “dx” into “du.”

-We form an easily integrable function and then integrate.

-Substitute all “x” back into the equation.

Finney Demana Homework

Antidifferentiation by Substitution

6.2 (Orange) 7.2 (Green/Yellow)

(17- 47 odd)

$$\int \cot x \; dx$$