Antiderivative

The opposite of the derivative.

Also referred to as the Indefinite Integral.

$$y = 3x^2$$

What is the antiderivative (indefinite integral) of this function?

What is a function, whose derivative will give me $y = 3x^2$?

$$\frac{d}{dx}[\ ?\ ] = 3x^2 \qquad ? = x^3 \qquad \boxed{\int 3x^2 = x^3}$$

What is the antiderivative (indefinite integral) of $y = x^4$?

What is a function, whose derivative will give me $y = x^4$?

$$\frac{d}{dx}[\ ?\ ] = x^4 \qquad ? = \frac{1}{5}x^5 \qquad \boxed{\int x^4 = \frac{1}{5}x^5}$$

$$\int 3x^4 =$$

$$\int 5x^{-3} =$$

General Power Rule for Integration

$$\int ax^n = \frac{a}{n+1}x^{n+1}$$

*Add one to the exponent (n) and then multiply the coefficient (a) by the reciprocal of the new exponent $\left(\frac{1}{n+1}\right)$.

$$\int 3x^2$$

$$\frac{d}{dx}[\ ?\ ] = 3x^2$$

$$x^3$$

But what about..... $x^3 + 7$

or $x^3 + 12$

or $x^3 - 5$

The correct answer is....

$\int 3x^2 = x^3 + C$, where C can be any constant.

Notation for Indefinite Integral

Variable of integration

Constant of integration

$$y = \int f(x)\,dx = g(x) + C.$$

Integrand

*You can think of $\int$ and dx as being like parenthesis (   ).

Integral Practice

$$\int x^4 + 3x - 2 \; dx =$$

$$\int \sin x \; dx \quad =$$

$$\int \cos x \; dx \quad =$$

$$\int dx =$$

Rewriting Before Differentiating

Original Integral	*Rewrite*	*Integrate*	*Simplify*
$\int \frac{1}{x^3}\,dx$	$\int x^{-3}\,dx$	$-\frac{1}{2}x^{-2} + C$	$-\frac{1}{2x^2} + C$
$\int \sqrt{x}\,dx$	$\int x^{1/2}\,dx$	$\frac{2}{3}x^{3/2} + C$	______
$\int 2\sin x\,dx$	$2\int \sin x\,dx$	$2(-\cos x) + C$	$-2\cos x + C$

General Solution

$$\int 3x^2 - 1\,dx = x^3 - x + C$$

$$g(x) = x^3 - x + C$$

is known as the General Solution.

Initial Condition and Particular Solution

If we are given an Initial Condition, then we can solve for a Particular Solution of the integral.

For example if we are asked to find the solution to $\int 3x^2 - 1\,dx$ that passes through the point (2,4).

$$g(x) = x^3 - x + C$$
$$4 = 2^3 - 2 + C$$
$$C = -2$$
$$g(x) = x^3 - x - 2$$

General Solution

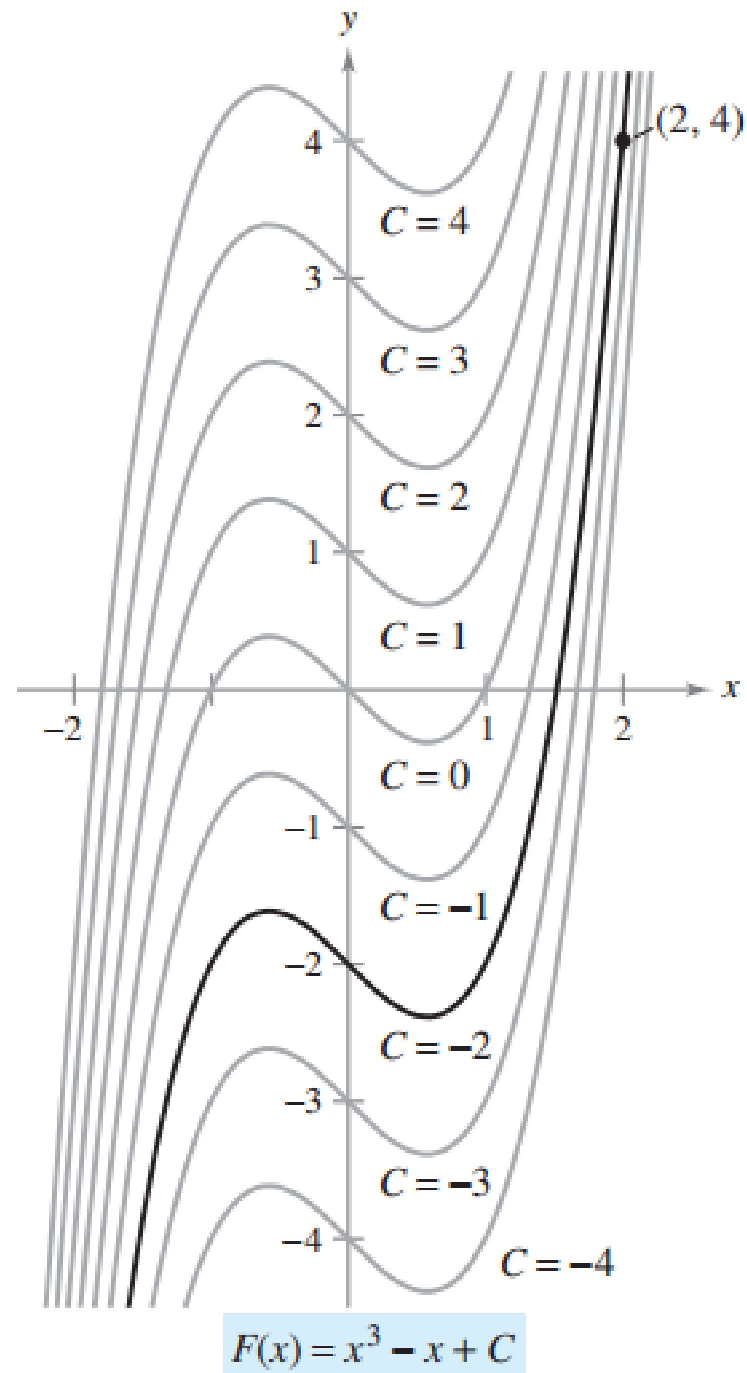
$$g(x) = x^3 - x + C$$

Initial Condition

$$(2,4)$$

Particular Solution

$$g(x) = x^3 - x - 2$$



Homework

4.1

P. 255 (1, 3, 15-41 odd)

Differential Equation

A differential equation (in x and y) is an equation that involves x, y and y' $\left(\frac{dy}{dx}\right)$.

Find the general solution of the differential equation: $y' = x^{5/4}$

$$\frac{dy}{dx} = x^{5/4}$$

$$dy = x^{5/4}\,dx$$

$$\int dy = \int x^{5/4}\,dx$$

$$\boxed{y = \frac{4}{9}x^{9/4} + C}$$

*Note that the derivative of the solution is the differential equation.

Vertical Motion Problem

A ball is thrown upward with an initial velocity of 64 ft/sec from an initial height of 80 feet. Use the fact that acceleration is -32 ft/s^2 to come up with a function for position $s(t)$.

Since acceleration is constant: $a(t) = s''(t) = -32$

$$\int s''(t)dt = \int -32\,dt$$

$$s'(t) = -32t + C_1 \qquad (s'(t) = velocity)$$

$$s'(0) = -32(0) + C_1 = 64 \qquad (Initial\ Velocity\ is\ 64)$$

$$C_1 = 64$$

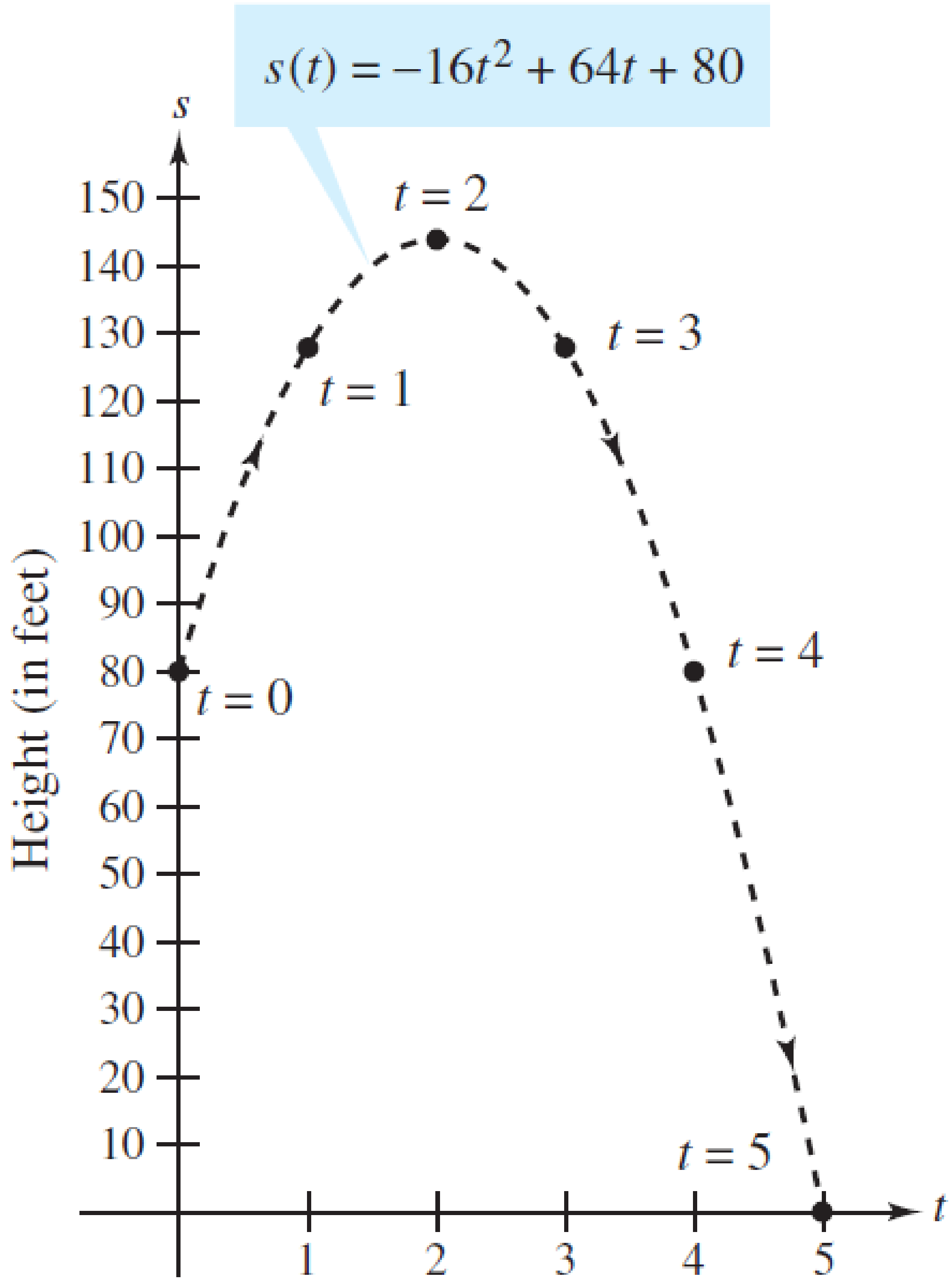
$$s'(t) = -32t + 64$$

$$s(t) = -16t^2 + 64t + C_2$$

$$s(0) = -16(0)^2 + 64(0) + C_2 = 80 \qquad (Initial\ Position\ is\ 80)$$

$$C_2 = 80$$

$$\boxed{s(t) = -16t^2 + 64t + 80}$$



$s(t) = -16t^2 + 64t + 80$
s
Height (in feet)
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
t = 0
t = 1
t = 2
t = 3
t = 4
t = 5
1
2
3
4
5
t

More Integration Practice

$$\int e^x \, dx$$

$$\int x^{-1} \, dx$$

More Integration Practice

$$\int x^2 + \sec^2 x \quad dx$$

$$\int \sqrt[3]{x}\,(x - 4) \; dx$$

$$\int \frac{\sin x}{\cos^2 x} \, dx$$

Homework

Section 4.1

Integration Worksheet

(6, 43, 45, 47, 48, 57, 61, 63, 65, 66, 75, 76)

(76 Hint: When Y=R, V=V_0)