Curve Sketching

Using all that we have learned about functions and their graph, can we sketch the curve?

Things to consider….

- Domain
- Intercepts (X and Y)
- Symmetry
- Asymptotes (Horizontal, Vertical and Slant)
- Extrema and Increasing/Decreasing Intervals
- Points of Inflection and Concavity

In order to guide the explanation of Curve Sketching, use the function

$$y = \frac{2x^2}{x^2 - 1}$$

Domain

- Is there anywhere where the denominator equals zero or anywhere the square root is a negative?

$$y = \frac{2x^2}{x^2 - 1}$$

x and y intercepts

$$y = \frac{2x^2}{x^2 - 1}$$

- Find x intercepts by letting y equal zero.

- Find y intercepts by letting x equal zero.

Symmetry

- A function is even, symmetrical about the $y - axis$, if for all x $f(-x) = f(x)$.

- A function is odd, symmetrical about the $origin$, if for all x, $f(-x) = -f(x)$.

$$f(x) = \frac{2x^2}{x^2 - 1}$$

Asymptotes

$$y = \frac{2x^2}{x^2 - 1}$$

Vertical Asymptote

If ever we evaluate a function and we get $\frac{\#}{0}$ then we know we have a Vertical Asymptote.

Horizontal Asymptote

$$\lim_{x \to -\infty} f(x) = L \quad \text{or} \quad \lim_{x \to \infty} f(x) = L$$

Vertical Asymptotes

$$\lim_{x \to 1^+} \frac{2x^2}{x^2 - 1} =$$

$$\lim_{x \to 1^-} \frac{2x^2}{x^2 - 1} =$$

$$\lim_{x \to -1^+} \frac{2x^2}{x^2 - 1} =$$

$$\lim_{x \to -1^-} \frac{2x^2}{x^2 - 1} =$$

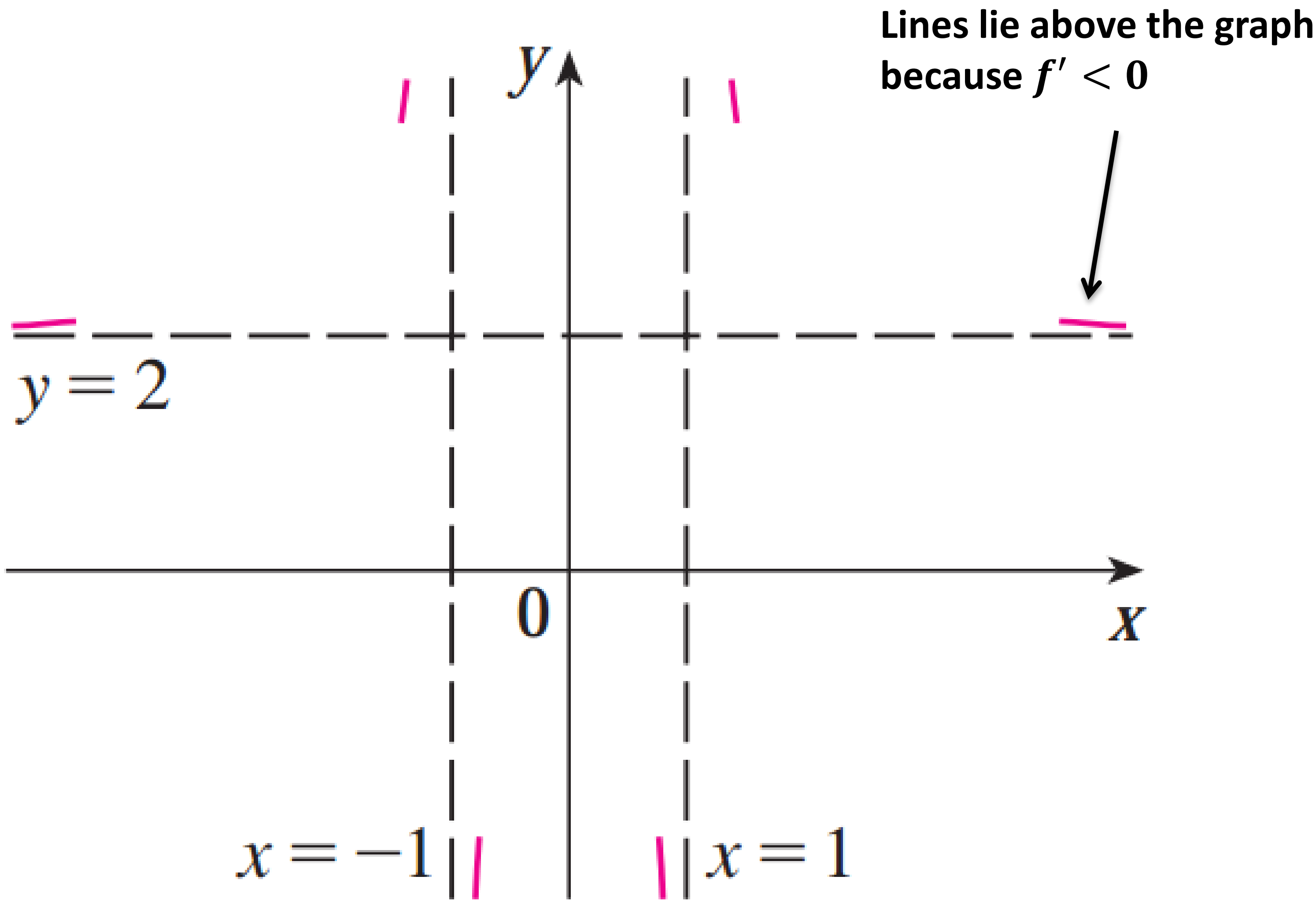
Extrema and Increasing/Decreasing Intervals

-Find all critical numbers where $f'(x) = 0$ or DNE

-Use a number line to evaluate where f is INC/DEC

Extrema and Increasing/Decreasing Intervals

-Use all critical numbers and where $f(x)$ DNE to construct a number line to evaluate where f is Increasing and Decreasing.



Lines lie above the graph
because $f' < 0$
y
$y = 2$
0
x
$x = -1$
$x = 1$

Points of Inflection and Concavity

-Find where $f'' = 0$ or DNE

-Use a number line to analyze concavity.

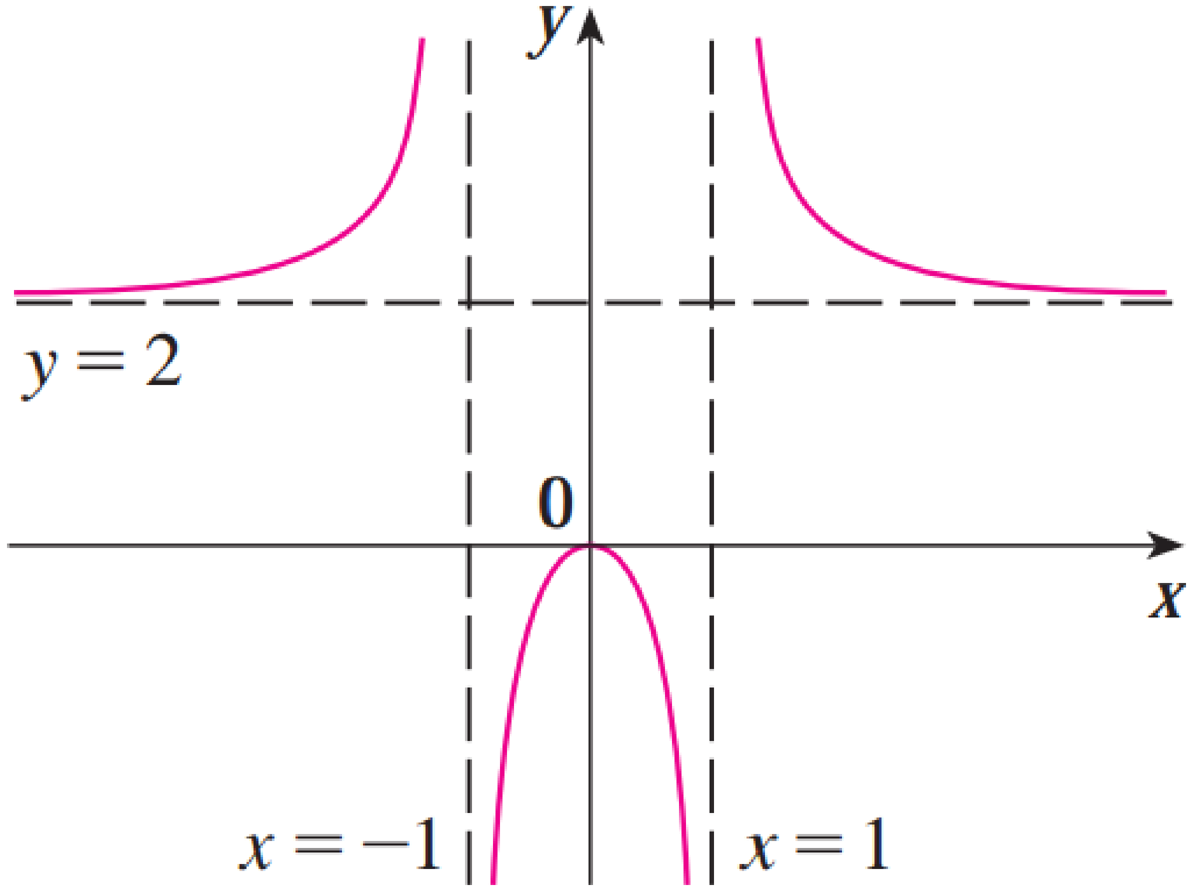
$$f(x) = \frac{2x^2}{x^2 - 1}$$

$$f''(x) = \frac{-4(x^2 - 1)^2 + 4x \cdot 2(x^2 - 1)2x}{(x^2 - 1)^4}$$

$$= \frac{12x^2 + 4}{(x^2 - 1)^3}$$

$$x = \pm 1$$

$$f(x) = \frac{2x^2}{x^2 - 1}$$



Slant Asymptotes

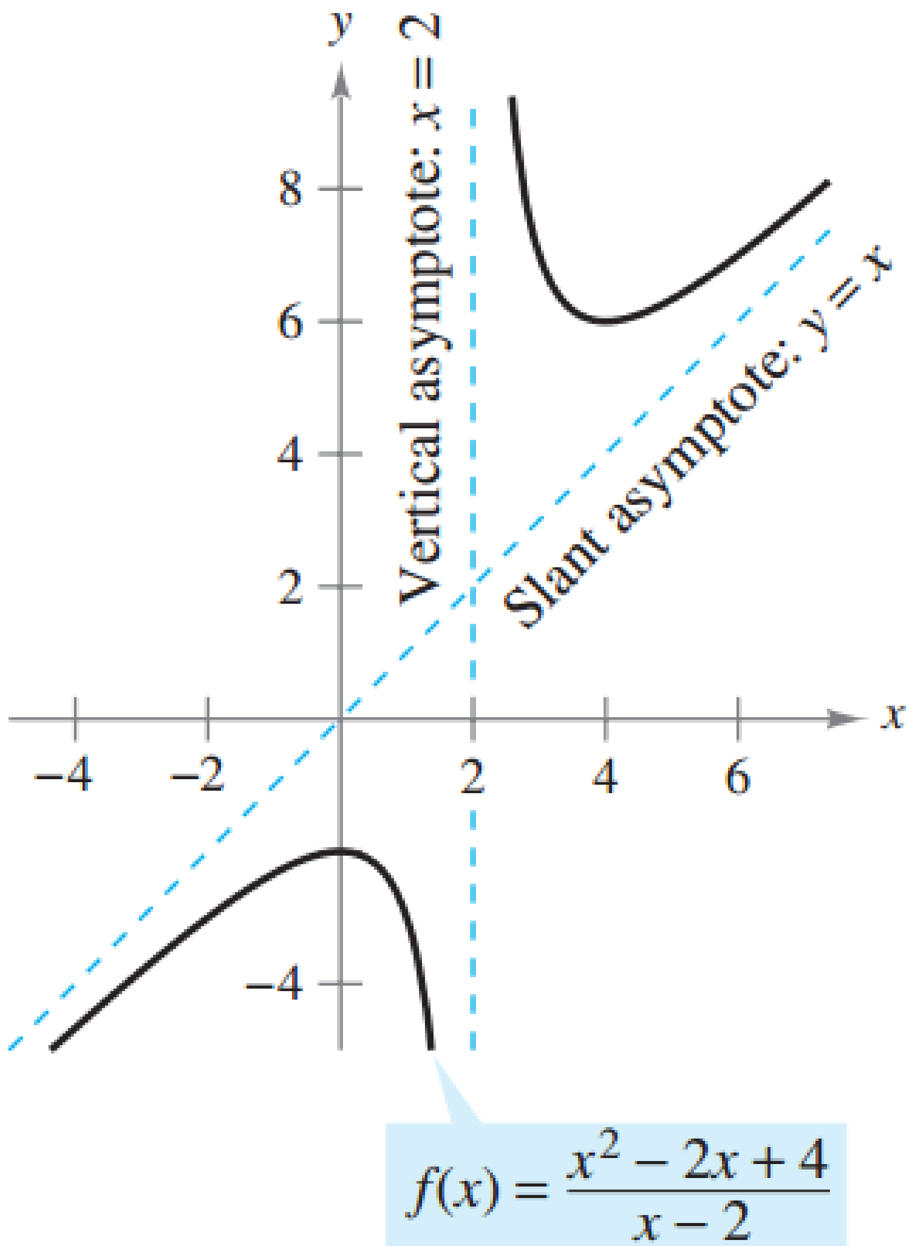
The graph of a rational function (having no common factors and whose denominator is of degree 1 or greater) has a slant asymptote if the degree of the numerator exceeds the degree of the denominator by exactly one.

$$f(x) = \frac{-3x^2 + 2}{x - 1}$$

$$f(x) = \frac{x^2 - 2x + 4}{x - 2}$$

$$f(x) = \frac{x^2 - 2x + 4}{x - 2}$$

In order to determine the slant asymptote, you must perform long division.


y
x
8
6
4
2
−4
−4
−2
2
4
6
Vertical asymptote: x = 2
Slant asymptote: y = x
$f(x) = \frac{x^2 - 2x + 4}{x - 2}$

$$f(x) = \frac{-3x^2 + 2}{x - 1}$$

Homework

Graph the following functions:

$$f(x) = \frac{x^2}{x^2 + 3}$$

$$f(x) = \frac{x^2 + 1}{x^2 - 9}$$

$$f(x) = \frac{x^2 - 6x + 12}{x - 4}$$

$$f(x) = \frac{2x^2 - 9}{x^2 - 4}$$

Sketch the curve of: $f(x) = \frac{2x^2-9}{x^2-4}$

Domain:

x and y intercepts:

$$f(x) = \frac{2x^2 - 9}{x^2 - 4}$$

Symmetry:

$$f(x) = \frac{2x^2 - 9}{x^2 - 4}$$

Asymptotes (Vertical or Horizontal)

$$f(x) = \frac{2x^2 - 9}{x^2 - 4}$$

Extrema and Intervals of Increasing/Decreasing

$$f(x) = \frac{2x^2 - 9}{x^2 - 4}$$

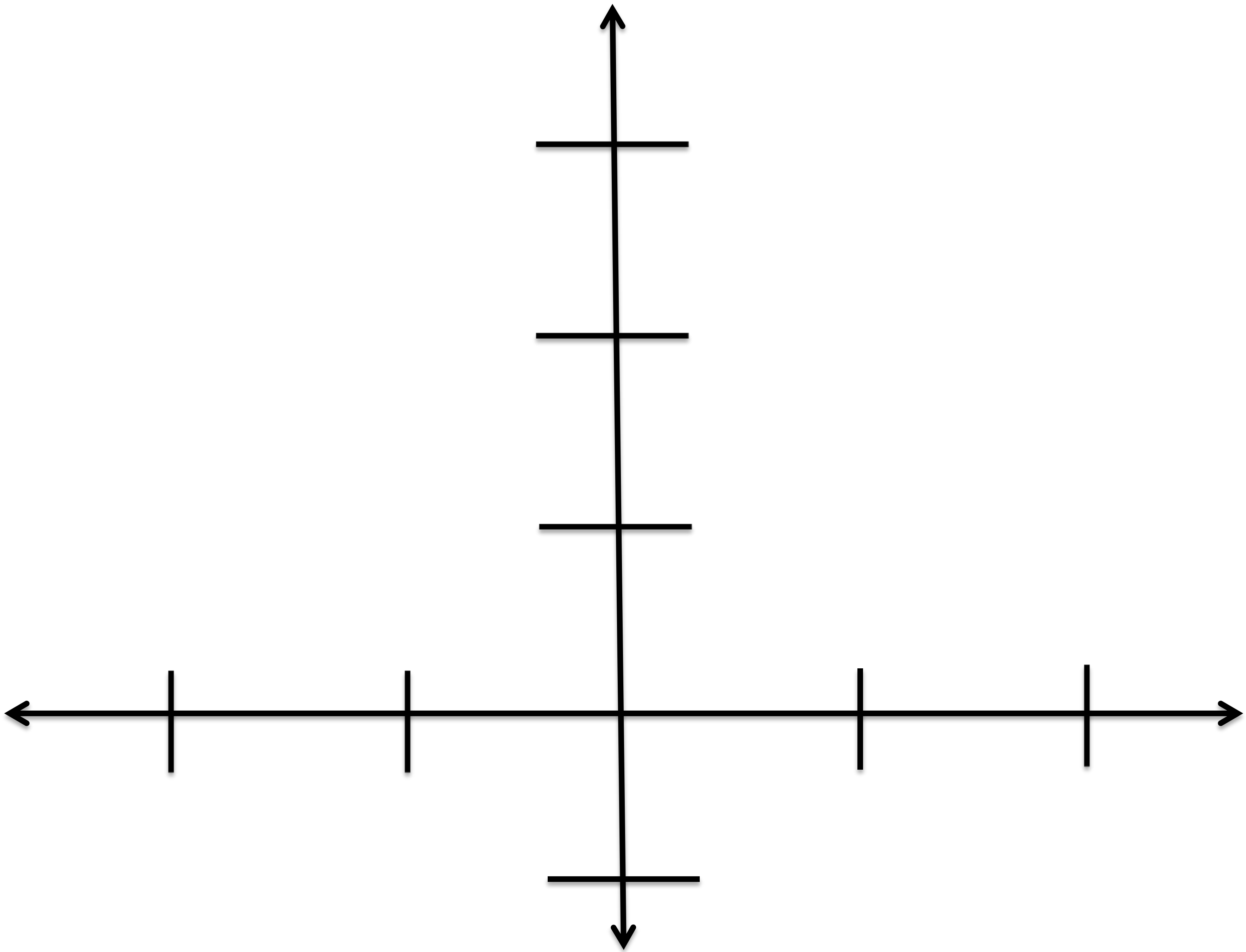
Extrema and Intervals of Increasing/Decreasing

$$f(x) = \frac{2x^2 - 9}{x^2 - 4}$$

Point of Inflection and Concavity

$$f(x) = \frac{2x^2 - 9}{x^2 - 4}$$

Point of Inflection and Concavity



Sketch the curve for: $f(x) = \frac{x^2}{\sqrt{x+1}}$

Domain:

X and Y Intercepts

$$f(x) = \frac{x^2}{\sqrt{x+1}}$$

Symmetry

$$f(x) = \frac{x^2}{\sqrt{x+1}}$$

Asymptotes

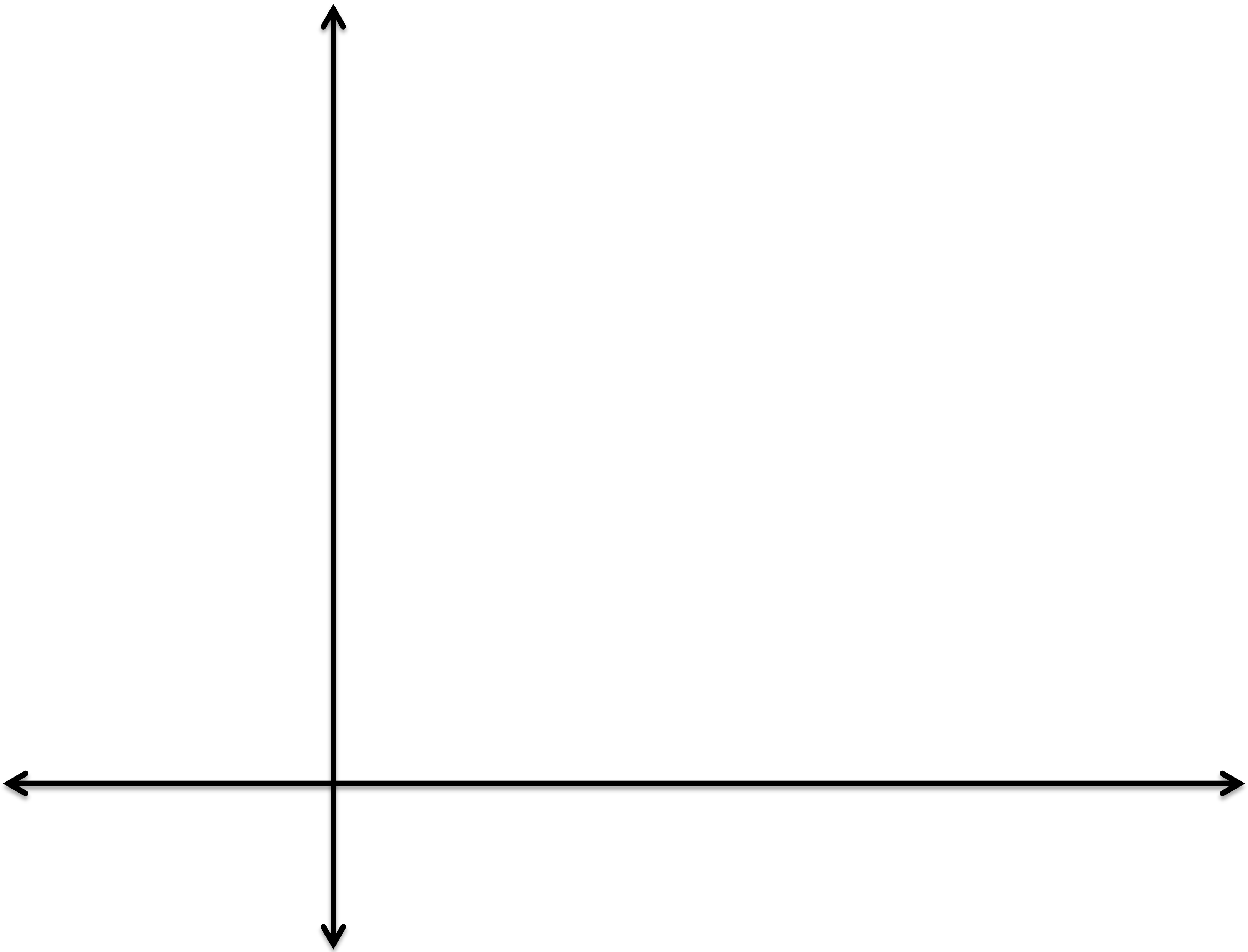
$$f(x) = \frac{x^2}{\sqrt{x+1}}$$

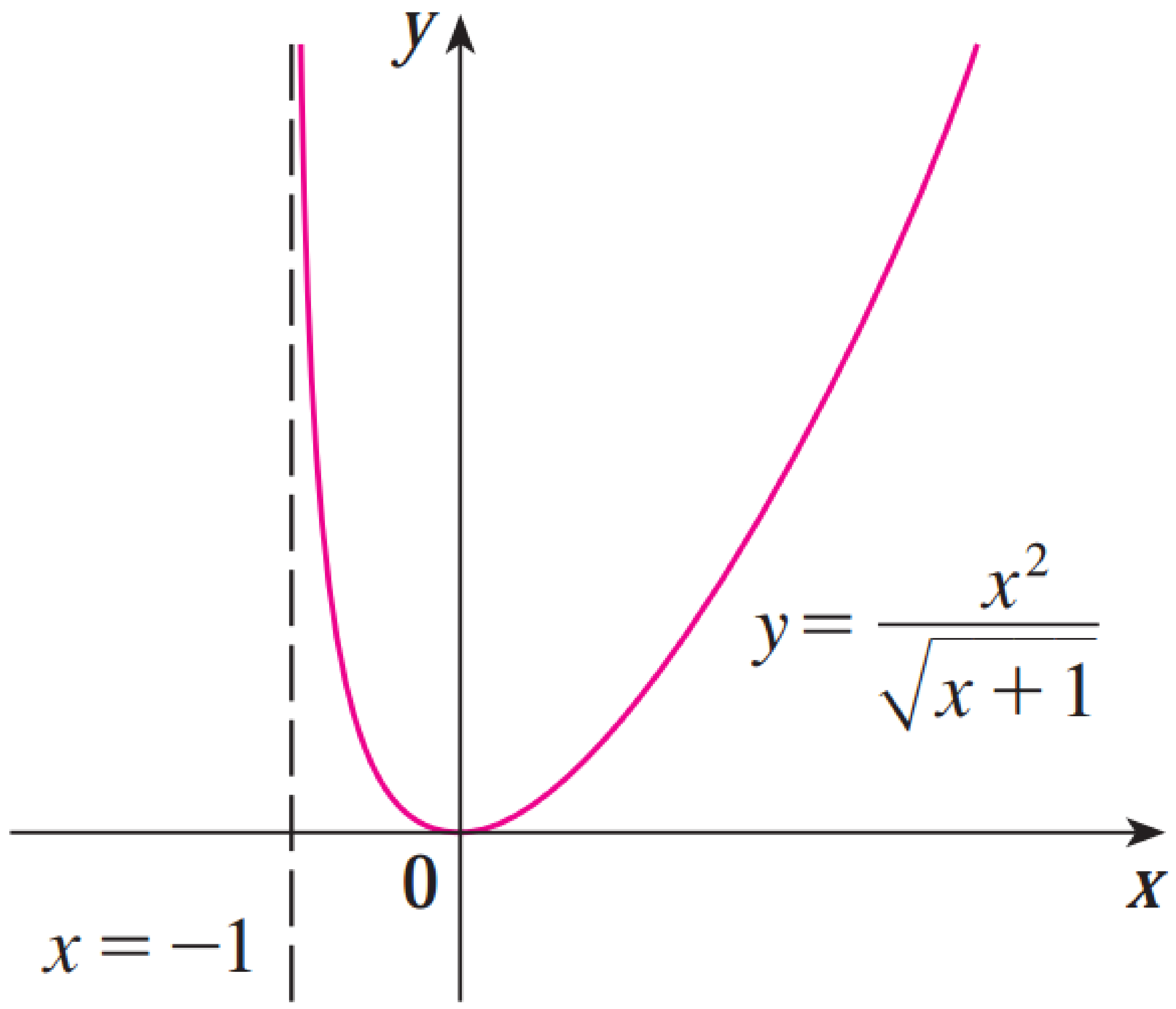
1st Derivative

$$f(x) = \frac{x^2}{\sqrt{x+1}}$$

2^{nd} Derivative

$$f(x) = \frac{x^2}{\sqrt{x+1}}$$




y
x
0
y = \frac{x^2}{\sqrt{x+1}}
x = −1