Connecting The Graphs of f, f', f''

1.

True or False If $f(c)$ is a local maximum of a continuous function f on an open interval (a, b), then $f'(c) = 0$. Justify your answer.

2. **Multiple Choice** Which of the following values is the absolute maximum of the function $f(x) = 4x - x^2 + 6$ on the interval $[0, 4]$?

(A) 0 (B) 2 (C) 4 (D) 6 (E) 10

3.

Multiple Choice If f is a continuous, decreasing function on $[0, 10]$ with a critical point at $(4, 2)$, which of the following statements *must be false*?

(A) $f(10)$ is an absolute minimum of f on $[0, 10]$.

(B) $f(4)$ is neither a relative maximum nor a relative minimum.

(C) $f'(4)$ does not exist.

(D) $f'(4) = 0$

(E) $f'(4) < 0$

4. **Multiple Choice** If an even function f with domain all real numbers has a local maximum at $x = a$, then $f(-a)$

(A) is a local minimum.

(B) is a local maximum.

(C) is both a local minimum and a local maximum.

(D) could be either a local minimum or a local maximum.

(E) is neither a local minimum nor a local maximum.

5. For some function, concavity is constant and $f'' = 3$. At the point $x = -2, f' = 5$. Find $f'(0)$ and $f'(-2.1)$.

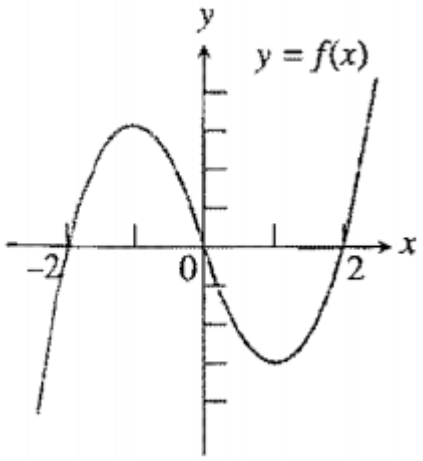
6.
True or False If $f''(c) = 0$, then $(c, f(c))$ is a point of inflection. Justify your answer.

7.
True or False If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a local maximum. Justify your answer.

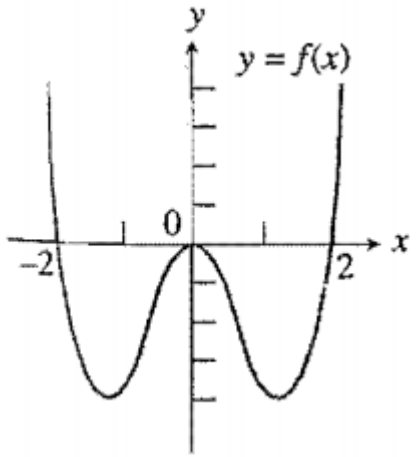
8. For some function, $f'' = 2x$. At the point $f'(3) = 4$. Which of the following are true? $f'(4) < 10$, $f'(4) > 10$ *or* $f'(4) = 10$. Justify your answer.

In Exercises 21 and 22, use the graph of the function f to estimate where **(a)** f' and **(b)** f'' are 0, positive, and negative.

21.

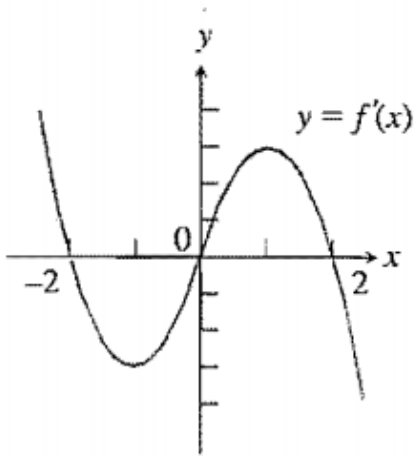


22.

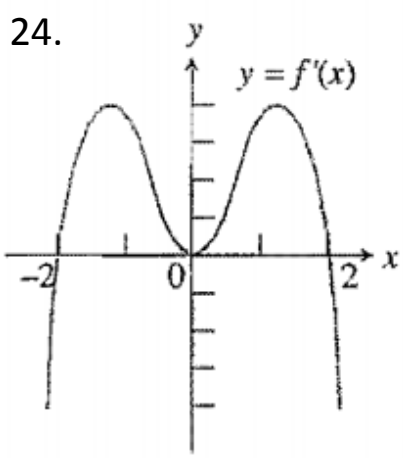


In exercises 23 and 24, tell me where f will be increasing and decreasing and where you will have any relative extrema. Also, tell me where f will be concave up and concave down and where you will have points of inflection.

23.



24.



25. Sketch a graph where the following are true.

$f(2) = f(4) = 0$

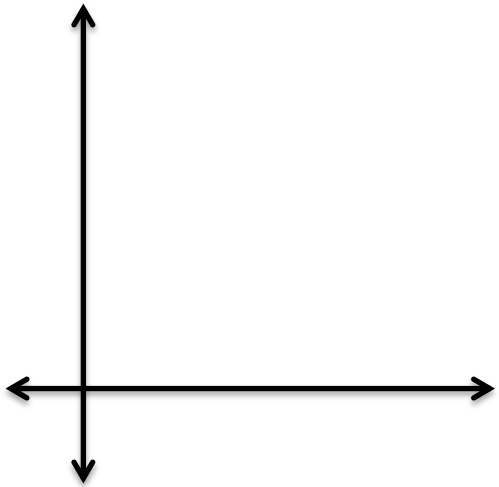
$f(3)$ is defined.

$f'(x) < 0$ if $x < 3$

$f'(3)$ does not exist.

$f'(x) > 0$ if $x > 3$

$f''(x) < 0,\ x \neq 3$



23. f increasing _____

f decreasing _____

Relative Extrema _____

Justify _____

_____

f is concave up _____

f is concave down _____

Points of Inflection _____

Justify _____

_____

24. f increasing _____

f decreasing _____

Relative Extrema _____

Justify _____

_____

f is concave up _____

f is concave down _____

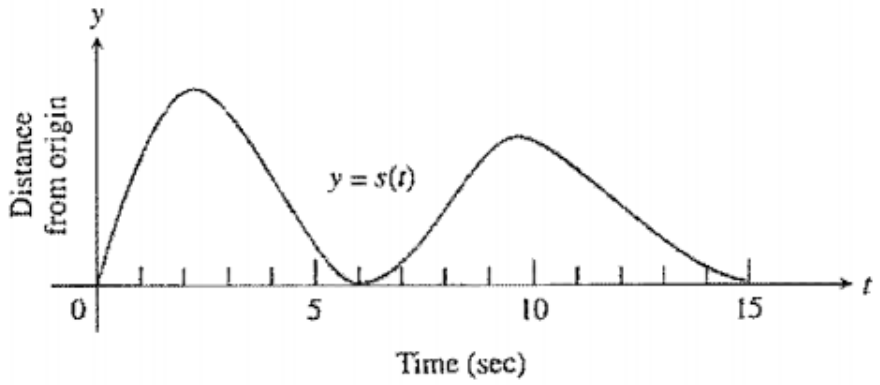
Points of Inflection _____

Justify _____

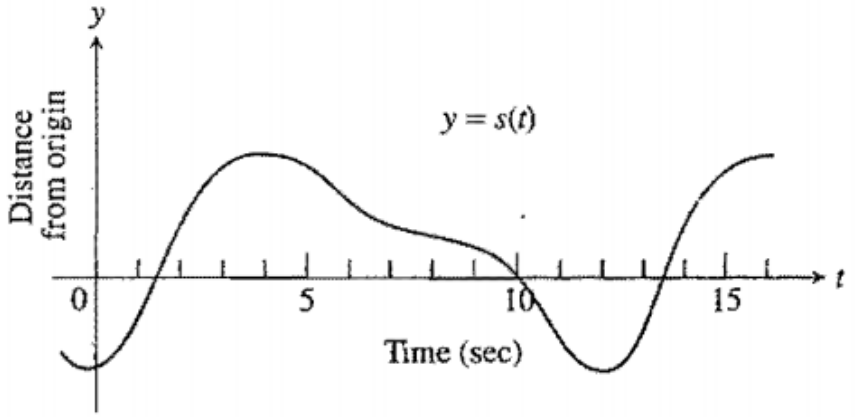
_____

In Exercises 29 and 30, the graph of the position function $y = s(t)$ of a particle moving along a line is given. At approximately what times is the particle's **(a)** velocity equal to zero? **(b)** acceleration equal to zero?

29.



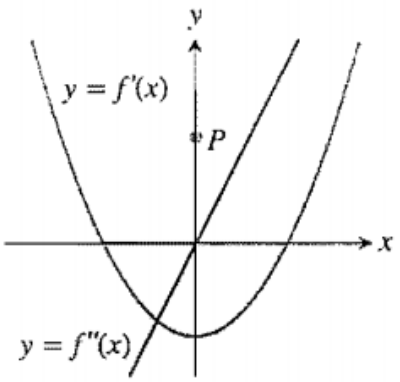
30.



Exercises 41 and 42 show the graphs of the first and second derivatives of a function $y = f(x)$. Copy the figure and add a sketch of a possible graph of f that passes through the point P.

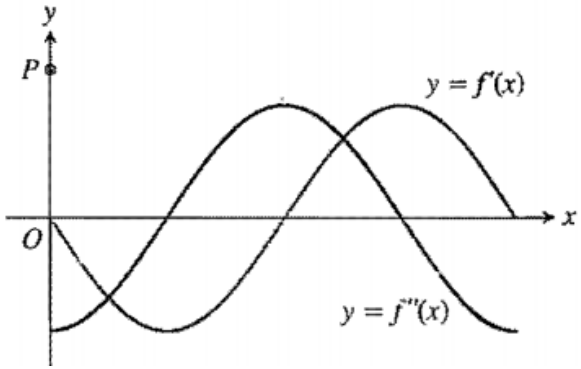
Also indicate the intervals of Increase/Decrease and Concavity to help you draw the graph.

41.



Inc:
Dec:
Conc. ↑:
Conc. ↓:

42.



Inc:
Dec:
Conc. ↑:
Conc. ↓:

Group Activity In Exercises 51 and 52, do the following.

(a) Find the absolute extrema of f and where they occur.

(b) Find any points of inflection.

(c) Sketch a possible graph of f.

51. f is continuous on $[0, 3]$ and satisfies the following.

x	0	1	2	3
f	0	2	0	-2
f'	3	0	does not exist	-3
f''	0	-1	does not exist	0

x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$
f	+	+	−
f'	+	−	−
f''	−	−	−

Group Activity In Exercises 51 and 52, do the following.

(a) Find the absolute extrema of f and where they occur.

(b) Find any points of inflection.

(c) Sketch a possible graph of f.

52. f is an even function, continuous on $[-3, 3]$, and satisfies the following.

x	0	1	2
f	2	0	-1
f'	does not exist	0	does not exist
f''	does not exist	0	does not exist

x	$0 < x < 1$	$1 < x < 2$	$2 < x < 3$
f	+	−	−
f'	−	−	+
f''	+	−	−

(d) What can you conclude about $f(3)$ and $f(-3)$?