The Second Derivative

What are the implications of the second derivative? What does it tell us about our function?

Definition of Concavity

The graph of a function f is **concave upward** if f' is increasing.

The graph of a function f is **concave downward** if f' is decreasing.

What does it mean if:
$y'' = 3$

- The function is concave up
- The slope is increasing
- What if the number was 2? 4? What does the 3 mean?

$$y' = \frac{d}{dx}[y] = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = 3 \qquad \Delta y = 3\Delta x$$

$$y'' = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d\left[\frac{dy}{dx}\right]}{dx} = \frac{\Delta\left[\frac{dy}{dx}\right]}{\Delta x} = 3$$

$$\Delta\left[\frac{dy}{dx}\right] = 3\Delta x$$

$$y'' = 3 \qquad \frac{d\left[\frac{dy}{dx}\right]}{dx} = 3$$

Concavity is constant. $y'' = 3$

The slope will be changing three times as much as x is changing.

$x = 2 \qquad y' = 5$

$x = 3 \qquad y' = 8$

$x = 2.1 \qquad y' = 5.3$

$x = 2.4 \qquad y' = 6.2$

$$y = \frac{3}{2}x^2 \qquad y'' = 3 \qquad \frac{d\left[\frac{dy}{dx}\right]}{dx} = 3$$

$$d\left[\frac{dy}{dx}\right] = 3dx$$

$x = 2 \quad y' = 6$

$x = 3 \quad y' =$

$x = 1 \quad y' =$

$x = 1.9 \quad y' =$

$x = -1 \quad y' =$

$$\frac{d\left[\frac{dy}{dx}\right]}{dx} = \frac{d^2y}{dx^2}$$

Estimating a Y-Value

Can you estimate $y(3)$?

$$y'(x) = 5x$$

$$y(2) = 4$$

Will $y(3)$ be more, less or equal to 14?

$$y(2.5) =$$

$$y(3) =$$

Estimating a Y-Value

$y'(x) = 5x$ $\qquad$ $y(2) = 4$

Will $y(3)$ be more, less or equal to 14?

Can you determine the actual value of $y(3)$ by finding the function $f(x)$?

$$y'(x) = 5x \qquad y(2) = 4$$

$$y(x) = \frac{5}{2}x^2 - 6$$

What if concavity is not a constant?

$$y''(x) = 2x$$
$$y'(3) = 5$$

Can you estimate the slope at $x = 4$?

$$y'' = \frac{d(y')}{dx} \qquad y'(4) =$$

At $x = 4$, will the slope be more, less or equal to 11?

$$y'(3.5) = \qquad y'(4) =$$

Can you determine the actual value of $y'(4)$?

$$y''(x) = 2x$$
$$y'(3) = 5$$