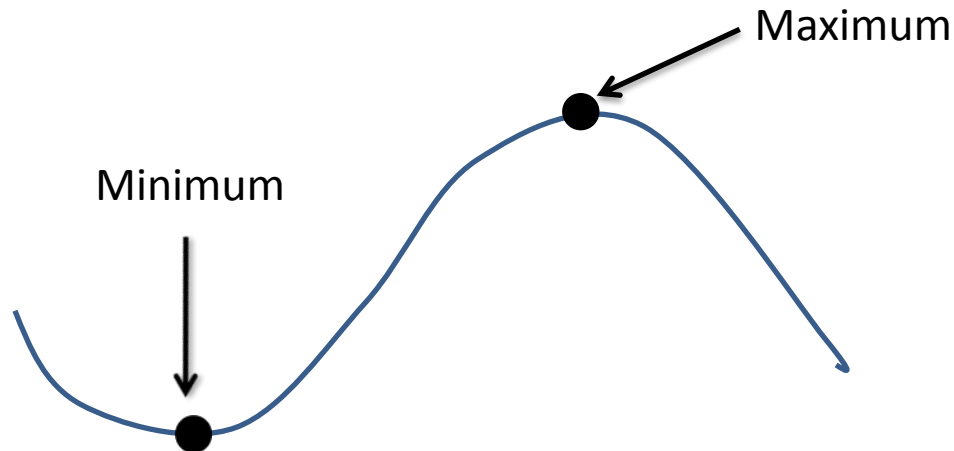
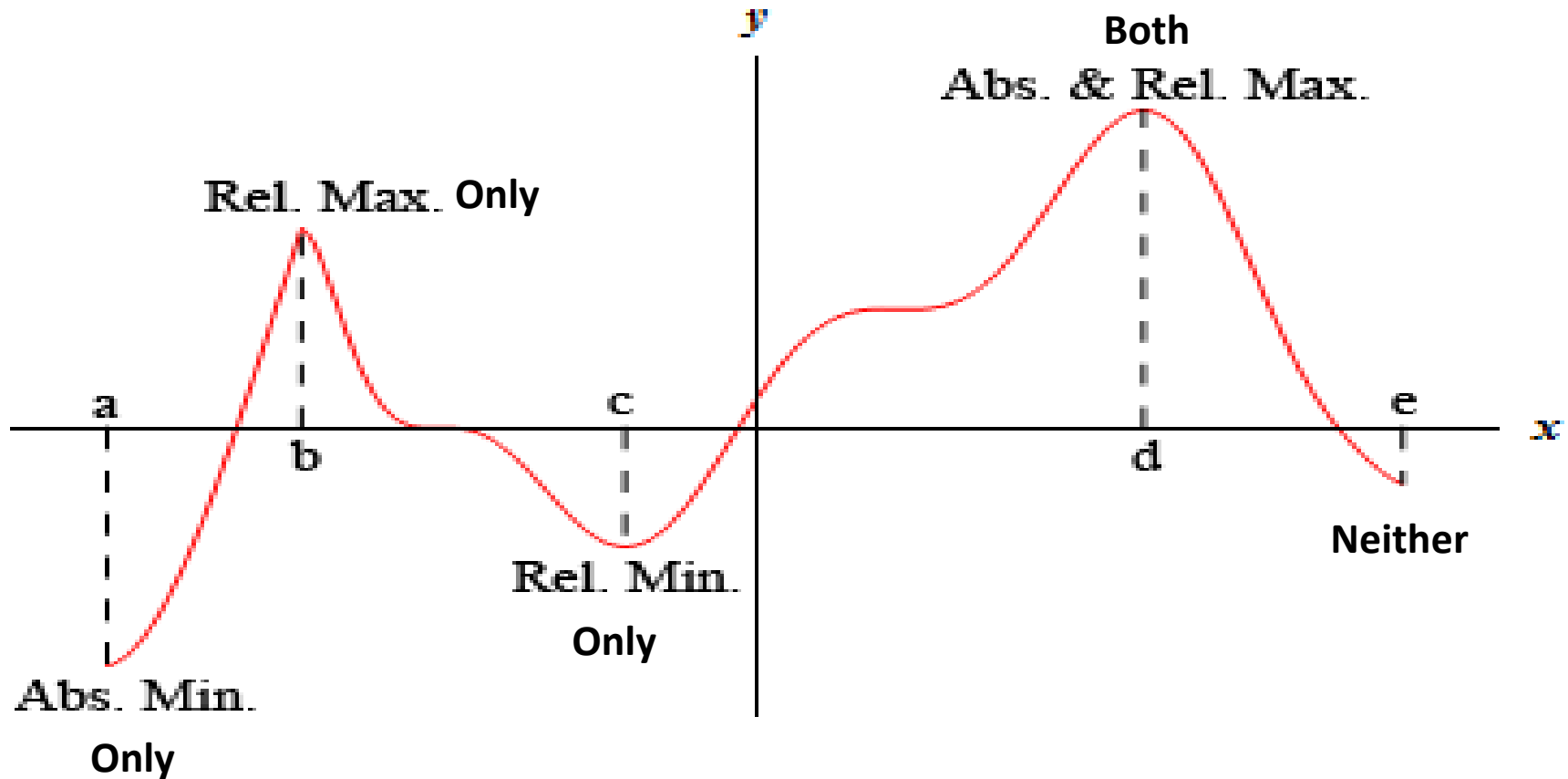


Extrema on an Interval

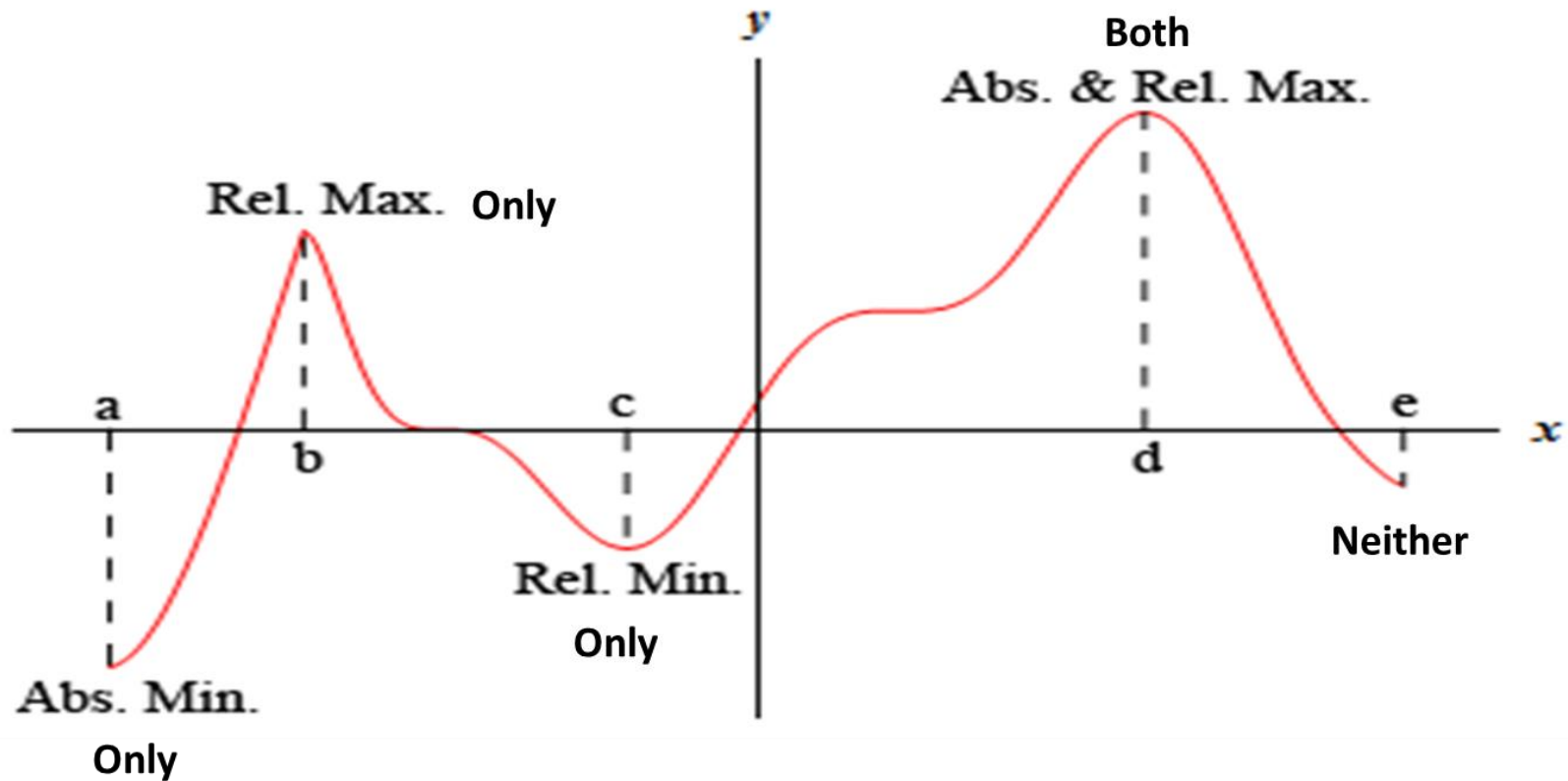
Where will we have Maximums and Minimums on a graph and how can we use Calculus to find these?



Relative vs Absolute Local vs Global



Based upon this picture, can you write the definition/difference between relative and absolute maximums and minimums?



An **Absolute** (global) Minimum or Maximum occurs over the entire interval. The Greatest or the Least of all.

A **Relative** (local) Minimum or Maximum occurs at a point **INSIDE** an interval. There must be points on the left and right. *Max or Min on an Open Interval

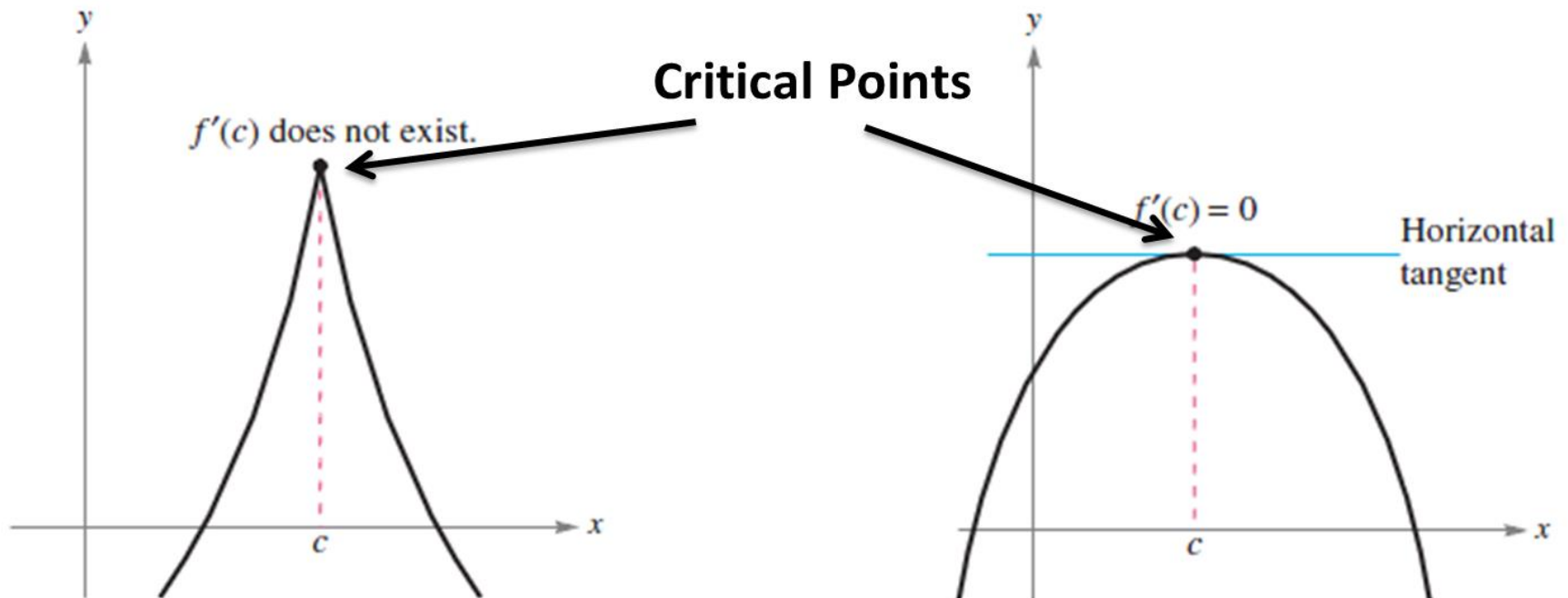
Critical Numbers

Critical Numbers are x -values where there could possibly be a Maximum or Minimum.

Let f be defined at c . The point $x = c$ is a critical number if:

$$f'(c) \text{ DNE}$$

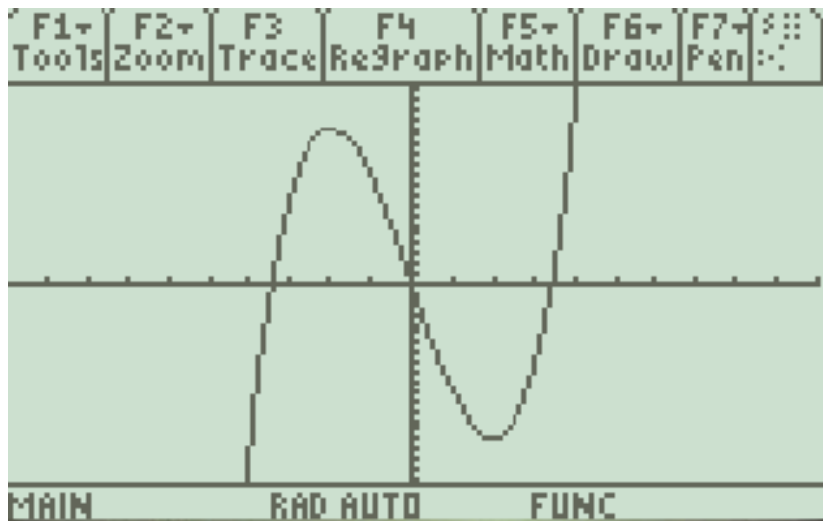
$$f'(c) = 0$$



For the following, tell me where you have a critical number. Then, graph the function with your calculator and tell me whether it is an absolute max/min, relative max/min or both.

$$f(x) = x^3 - 12x$$

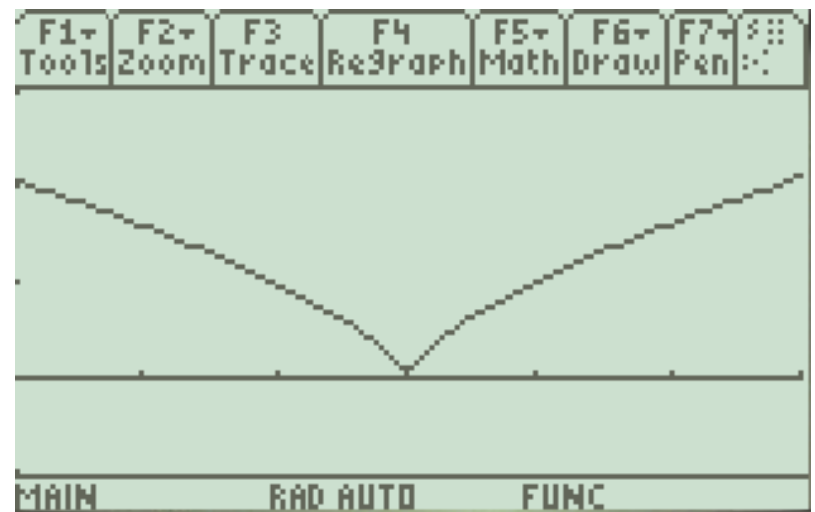
Critical Points: $x = -2$
 $x = 2$



Relative Max and Relative Min

$$f(x) = (x - 3)^{\frac{2}{3}}$$

Critical Points: $x = 3$

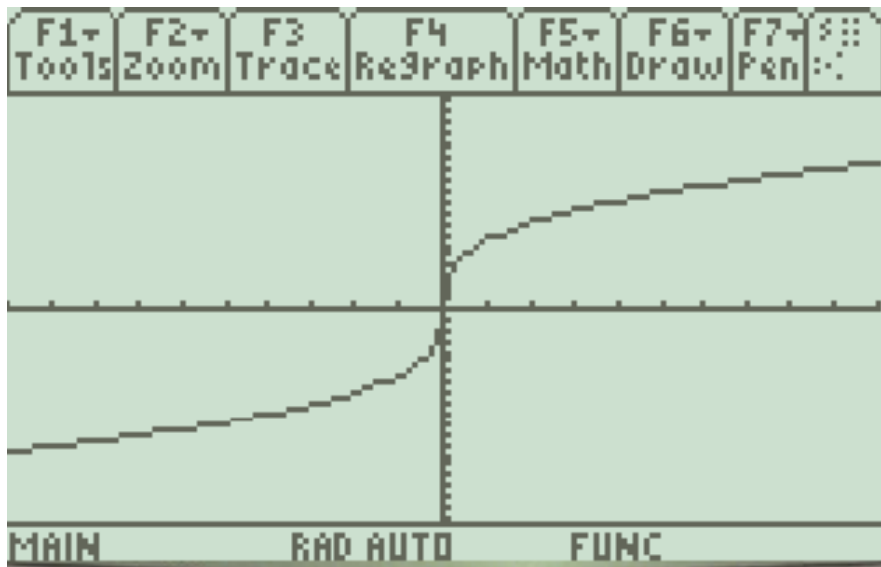


Absolute and Relative Min

For the following, tell me where you have a critical number. Then, graph the function with your calculator and tell me whether it is an abs/rel max/min.

$$f(x) = \sqrt[3]{x}$$

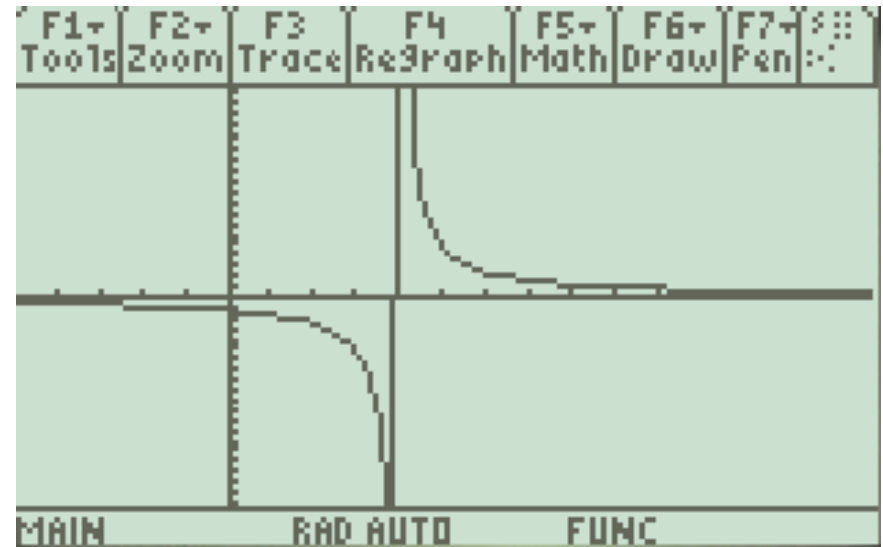
Critical Point: $x = 0$



No max/min

$$f(x) = \frac{5}{x-4}$$

No Critical Points!!



*** $f(c)$ must exist in order for c to be a critical point.**

Theorem 3.2: If f has a relative minimum or relative maximum at $x = c$, then c is a critical number of f .

Will the following always hold True?

If c is a critical number of f , then f has a relative minimum or relative maximum at $x = c$.

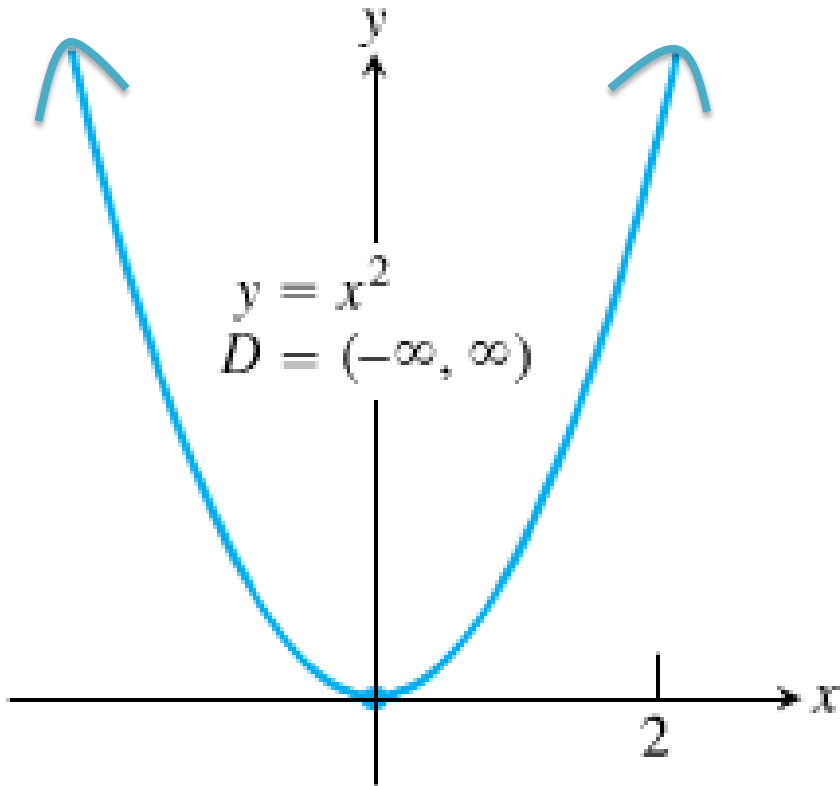
If c is not a critical number of f , then f does not have a relative minimum or relative maximum at $x = c$.

Finding Absolute Extrema on a Closed Interval

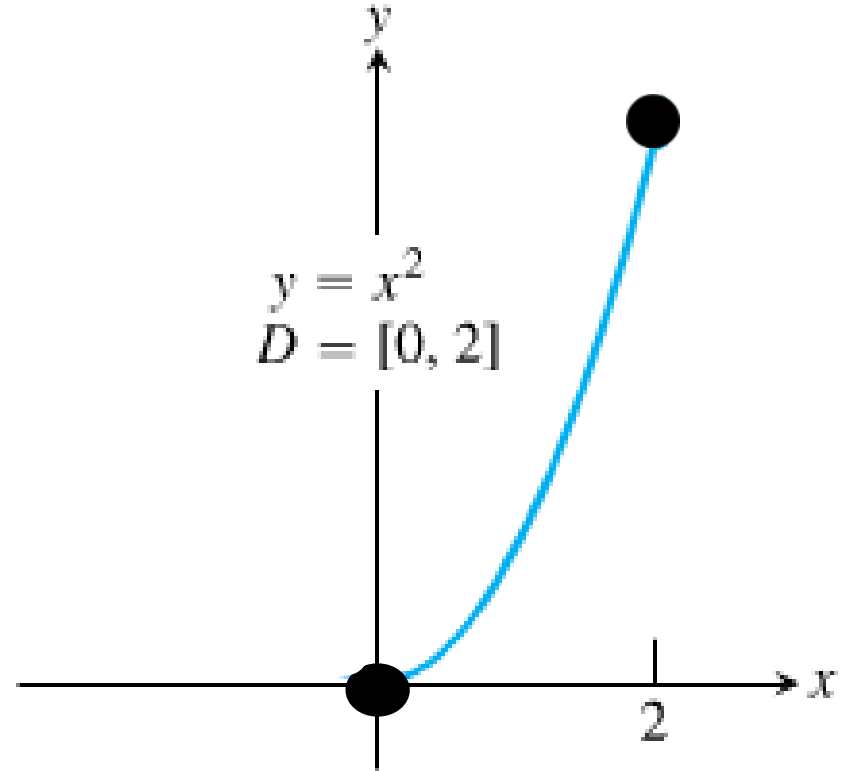
If given a closed interval, how do we find the absolute maximum and the absolute minimum?

Will there always be an absolute maximum and an absolute minimum?

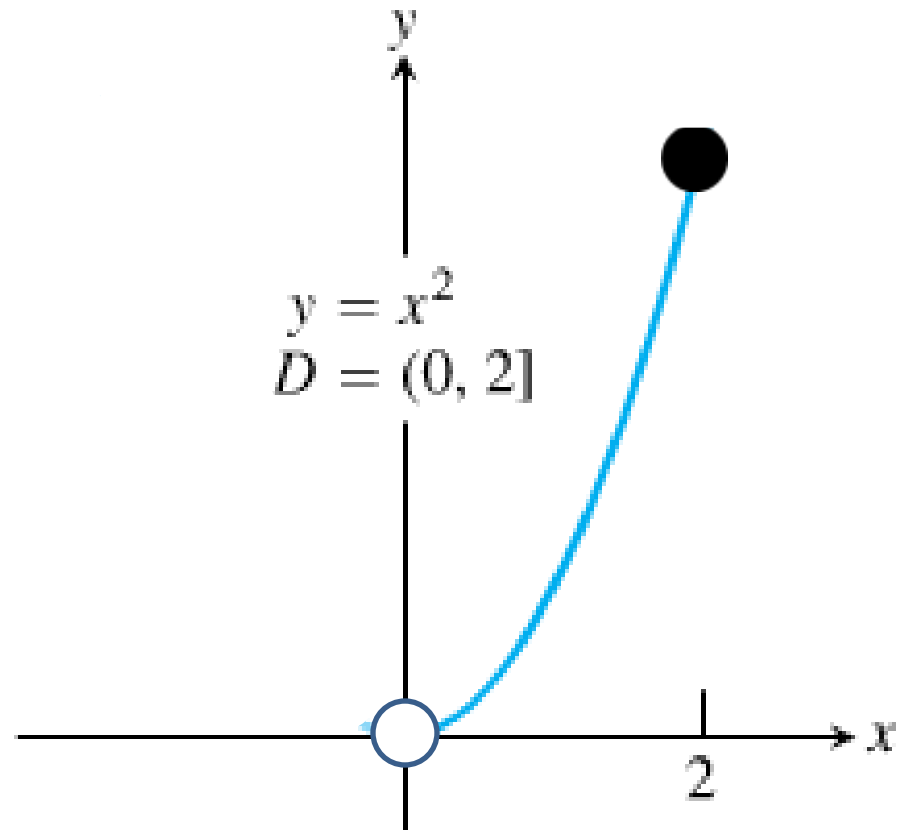
Open vs. Closed intervals.



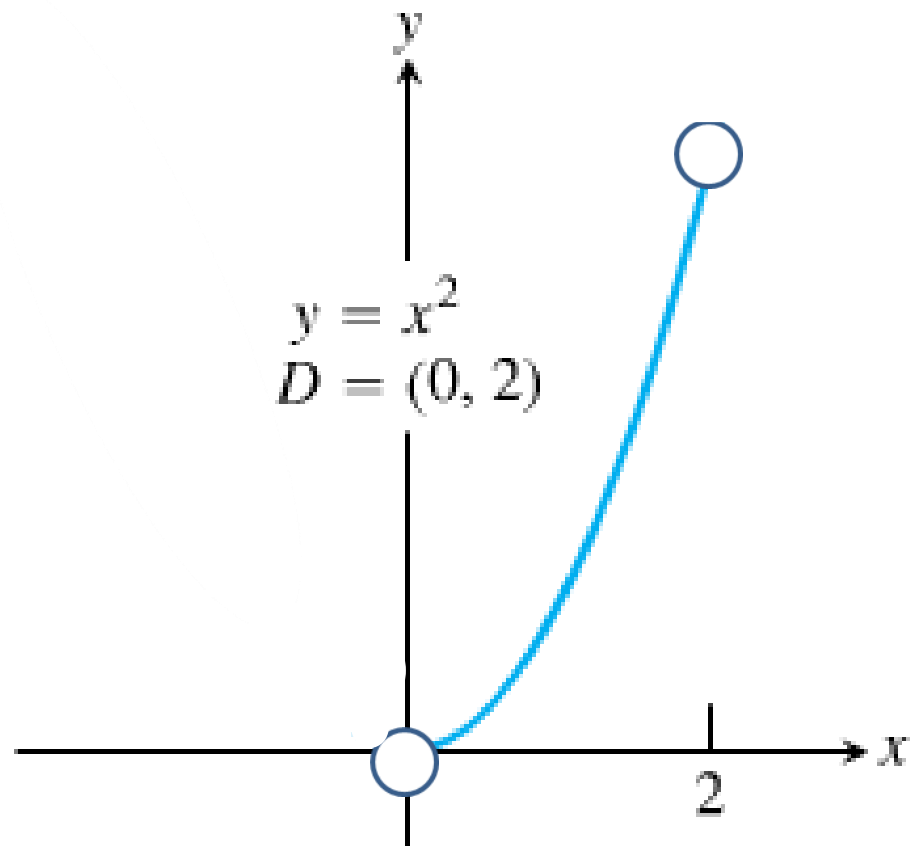
Both an absolute and relative minimum.



Absolute Max **ONLY**
Absolute Min **ONLY**



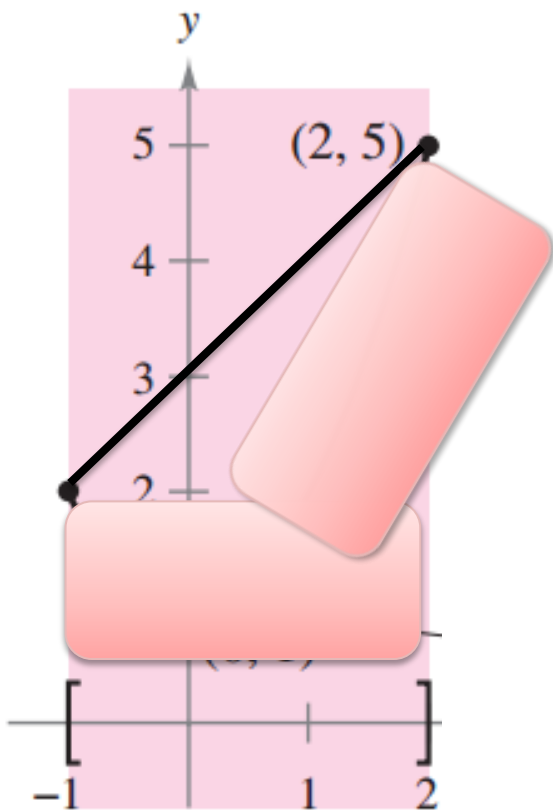
Absolute Maximum **Only**



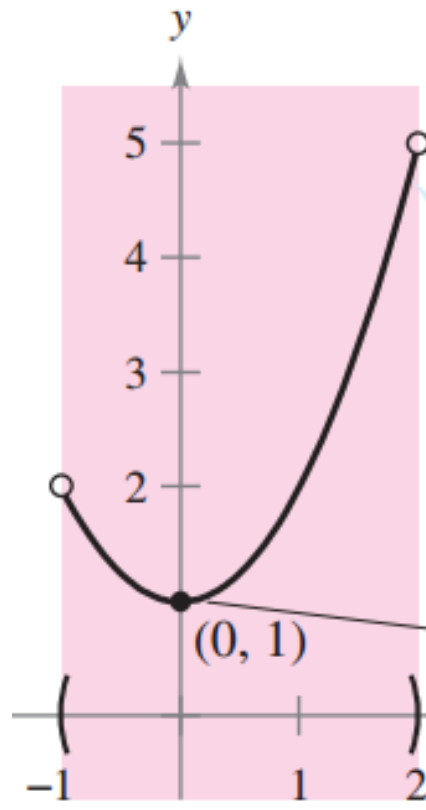
No Maximums or Minimums

Extreme Value Theorem

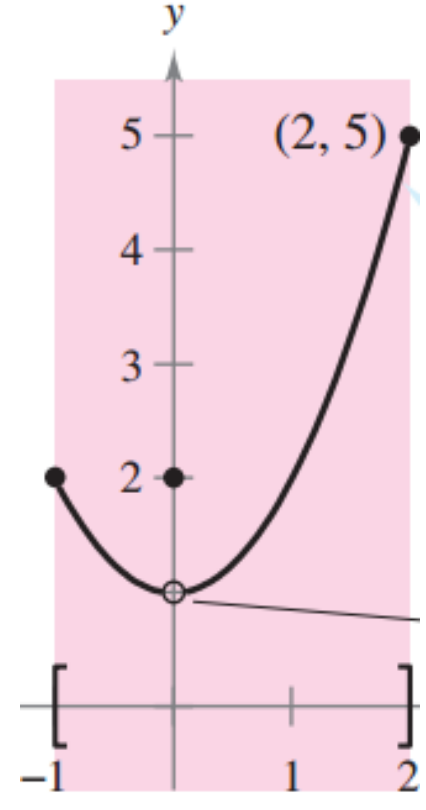
All **Continuous Closed** Intervals will **always** have both an **absolute** maximum and an **absolute** minimum.



Both Min and Max



Min only



Max only

Finding Extrema

- When finding **absolute extrema** on a **continuous and closed interval**, we must look at all **critical points** and all **endpoints**.
- You must plug in all critical points and endpoints to the original function in order to find the largest and smallest y -values.

EXAMPLE: Find all absolute extrema for the following function and interval:

$$g(t) = 2t^3 + 3t^2 - 12t + 4 \quad \text{on} \quad [-4, 2]$$

$$g'(t) = 6t^2 + 6t - 12$$

$$0 = 6(t + 2)(t - 1)$$

We have critical points at: $t = -2, 1$

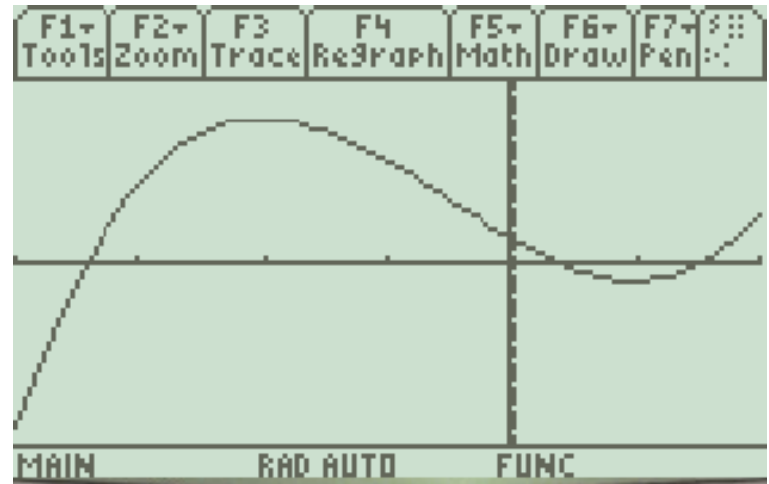
We must also consider our endpoints as they are also possible extrema.

We have endpoints at: $t = -4, 2$

***Now we should evaluate the function $g(t)$ at each of these to determine if we have any absolute extrema.**

- Critical Points: -2, 1
- Endpoints: -4, 2

$$g(t) = 2t^3 + 3t^2 - 12t + 4$$



- $g(-2) = 24$
- $g(1) = -3$
- $g(-4) = -28$
- $g(2) = 8$

We can thus see that our absolute maximum occurs at $t = -2$ and is 24 (a critical point).

Our absolute minimum occurs at $t = -4$ and is -28 (an endpoint).

TI~*n*spire Window Edit Instructions

After Graphing your function

Press Menu > 4: Window/Zoom > 1: Window Settings

Xmin: -4

Xmax: 2

Ymin: -30

Ymax: 30

TI-89 Window Edit Instructions

After Graphing your function

Press Green Diamond > F2 (Window)

xmin: -4

xmax: 2

yscl: 1

ymin: -30

ymax: 30

yscl: 1

Finding Absolute Maximums and Minimums of a Function $f(x)$ on a closed interval

$f(x)$ on a Closed Interval

$[a, b]$



Find Critical Point

$f'(x) = 0$ or $f'(x)$ DNE



Plug in Critical
Points to $f(x)$



Plug in Endpoints
to $f(x)$



The largest is the absolute maximum, the smallest is the absolute minimum.

Critical Points

*$f'(x) = 0$ or
 $f'(x)$ DNE*

**Relative Maximum
or Minimum**

Endpoints

**Absolute
Maximum
or Minimum**

Assignment

Extrema 1 Worksheet

(1-11 odd, 2, 13, 15, 17)

(Actually take the derivative in 3-8, what does the derivative of the point in question equal?)

(23-39 every other odd, 53, 55, 57)