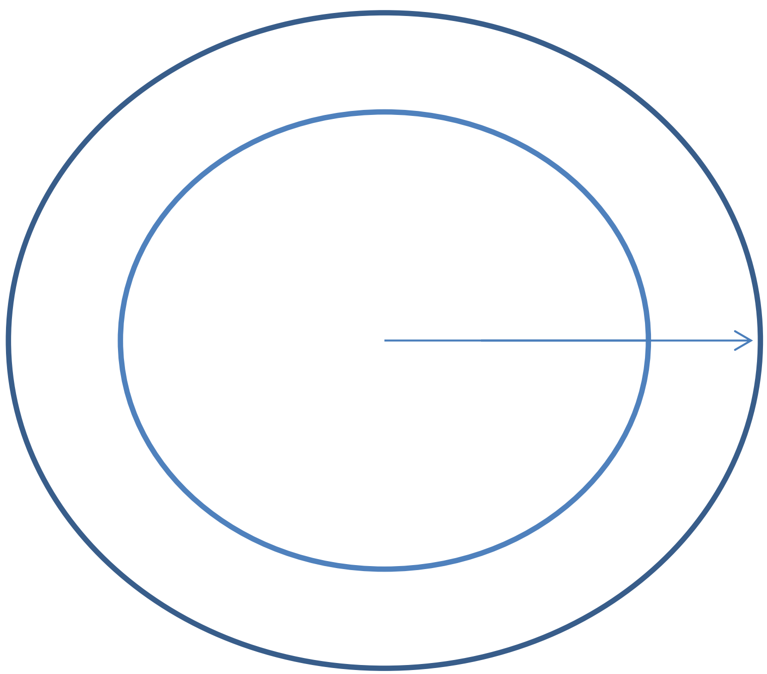
RELATED RATES

How fast or slow something is changing with respect to time.

How are two rates related to one another?



Related Rates

- **Rate** – how fast or slow something changes over time.

Velocity = 5 m/s $$\frac{\Delta position}{\Delta time} = \frac{ds}{dt} = 5\ m/s$$

- Rates are usually "with respect to" **time**
- Change in Y with respect to x… $\boldsymbol{\frac{dy}{dx}}$
- Change in Y with respect to time (t) ….. $\boldsymbol{\frac{dy}{dt}}$
- Change in X with respect to time (t) ….. $\boldsymbol{\frac{dx}{dt}}$

$$\frac{d}{dx}[y(x)]^2 =$$

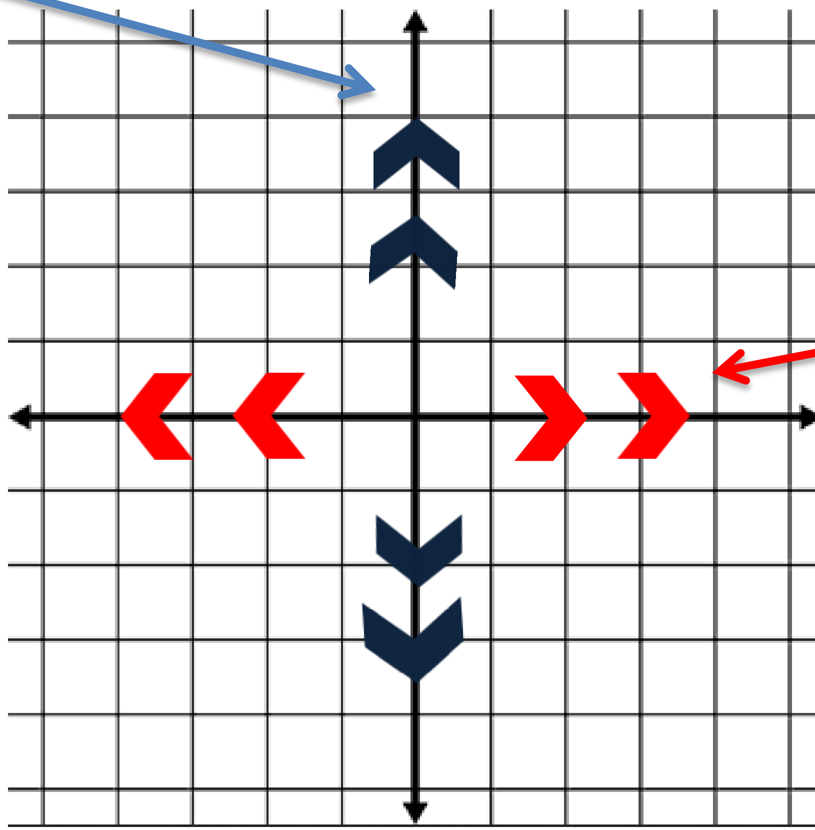
$$\frac{d}{dt}[y(t)]^2 =$$

$$\frac{d}{dt}[x(t)]^2 =$$

$$\frac{dx}{dt} \quad and \quad \frac{dy}{dt}$$

$\frac{dy}{dt}$

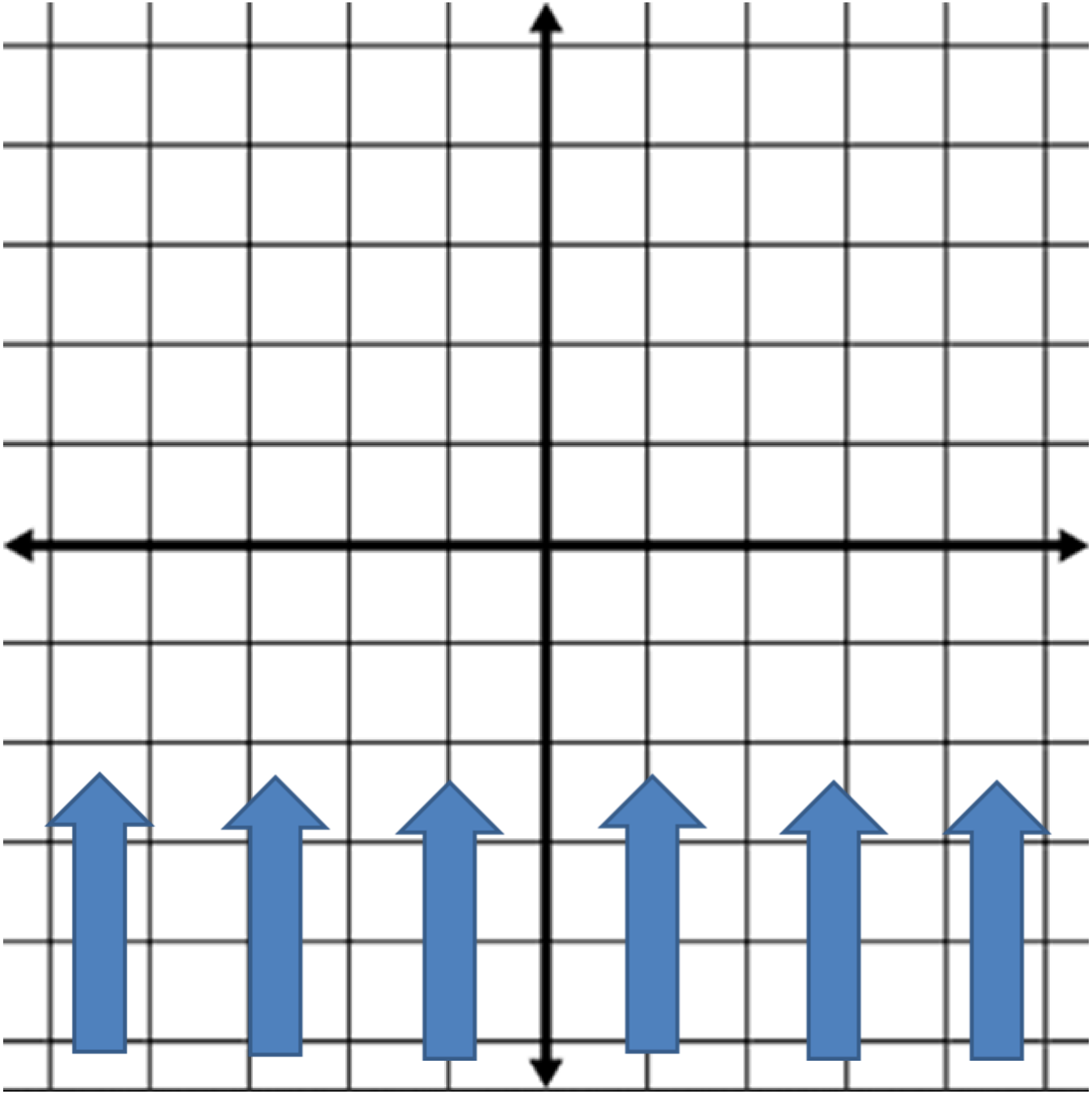
The rate at which a Particle is moving Up (+) or down (-).



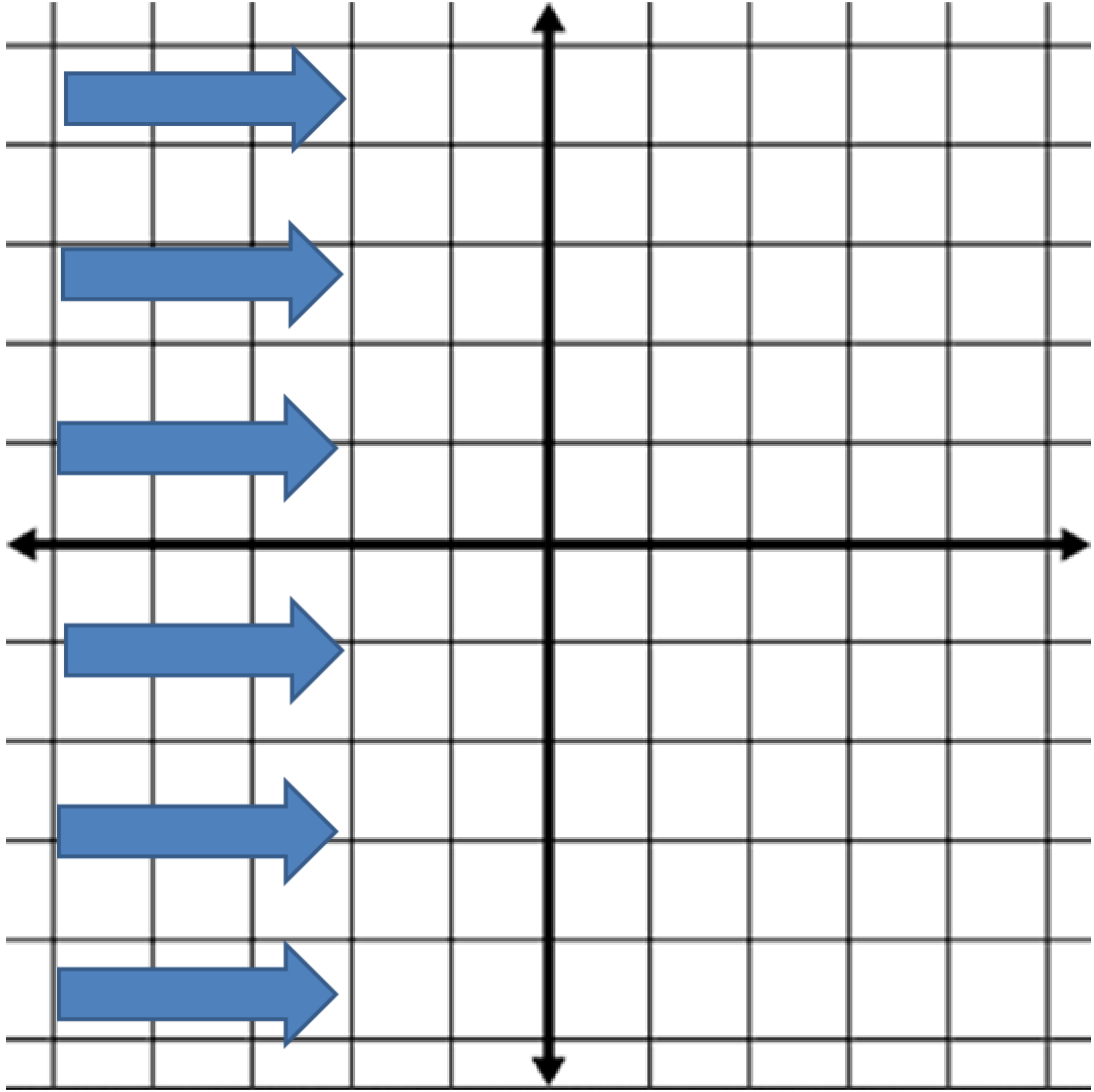
$\frac{dx}{dt}$

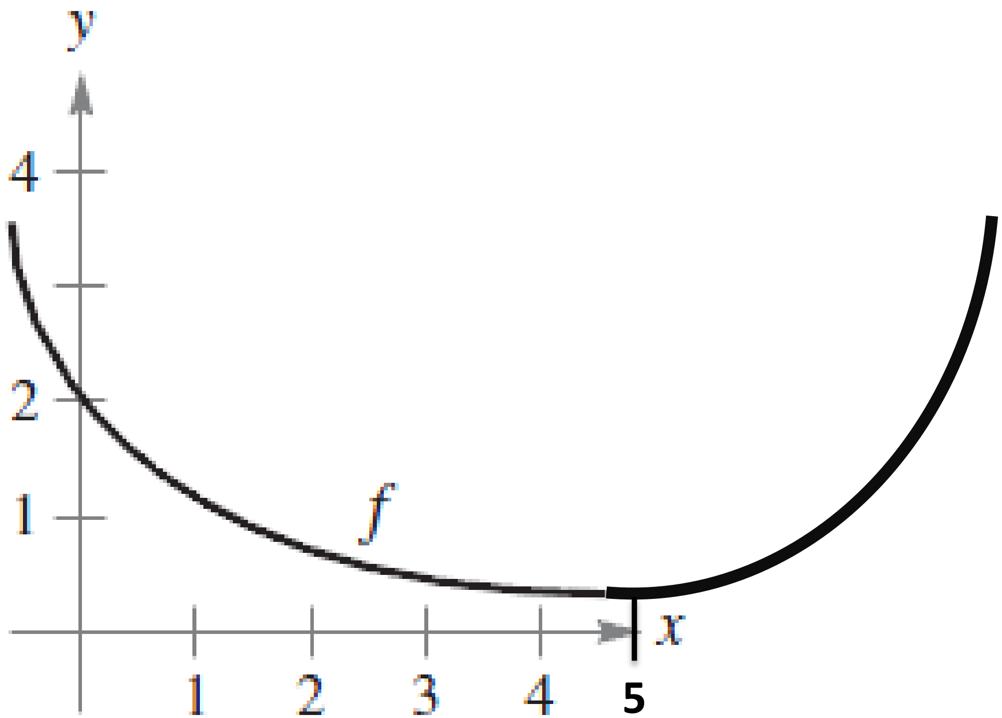
The rate at which a Particle is moving to The right (+) or left (-).

$$\frac{dx}{dt} = 2$$



$$\frac{dy}{dt} = 2$$



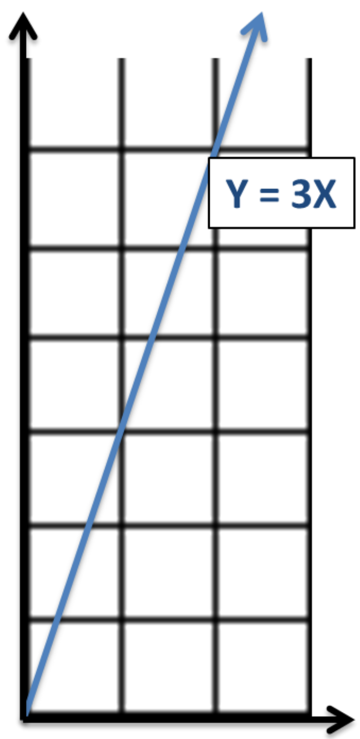


If $\frac{dx}{dt}$ is positive, Is $\frac{dy}{dt}$ positive or negative?

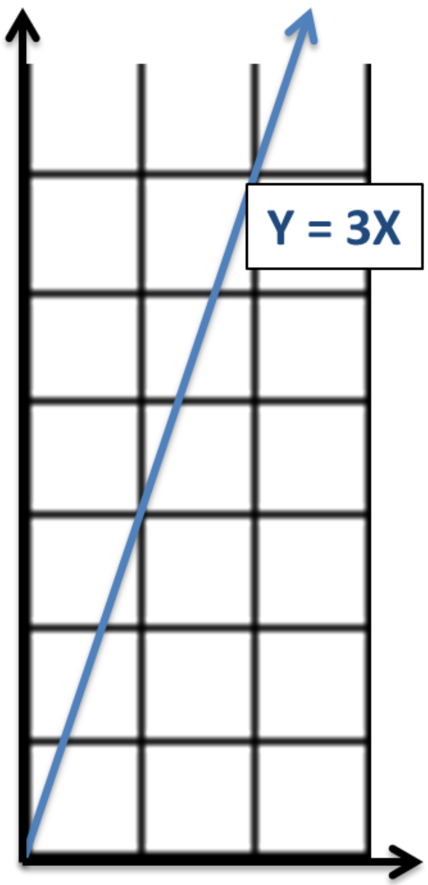
If $\frac{dx}{dt}$ is negative, Is $\frac{dy}{dt}$ positive or negative?

Finding Rates of Change $\left(\frac{dy}{dt}\right)$ or $\left(\frac{dx}{dt}\right)$

A particle is traveling along the line Y = 3X at a **horizontal** rate of 2 units/second $\left(\frac{dx}{dt} = 2\right)$.



Find the vertical rate of change of the particle at the point (2, 6).



If a particle is traveling horizontally at a rate of 2 units/sec.

$$\frac{dx}{dt} = 2$$

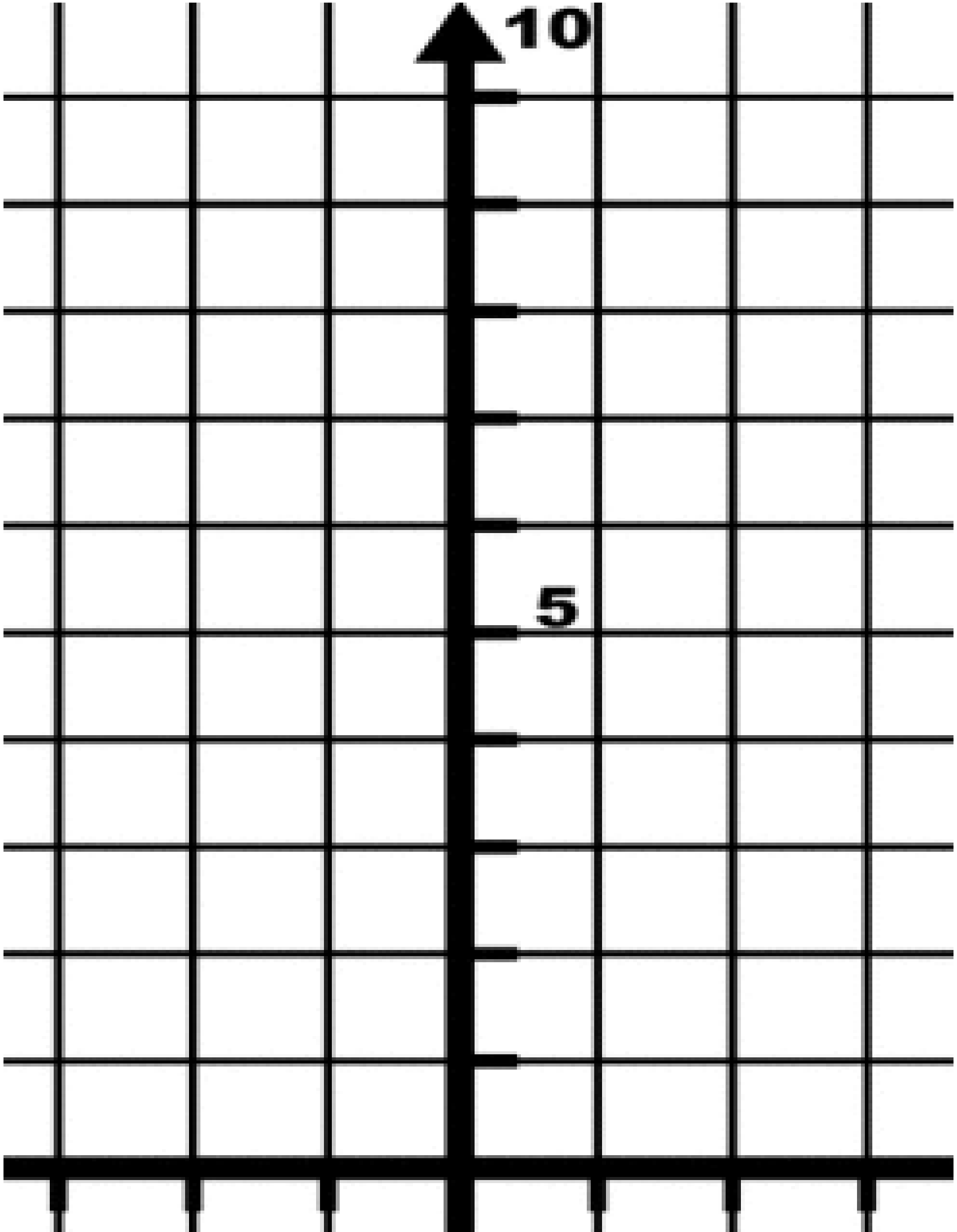
It must be traveling vertically at a rate of ____ units/sec.

$$\frac{dy}{dt} = \_\_\_\_ \frac{units}{sec}$$

Now using calculus:

$$\frac{d}{dt}[y = 3x]$$

$$\frac{d}{dt}[y(t) = 3x(t)]$$



$$y = x^2 + 1$$

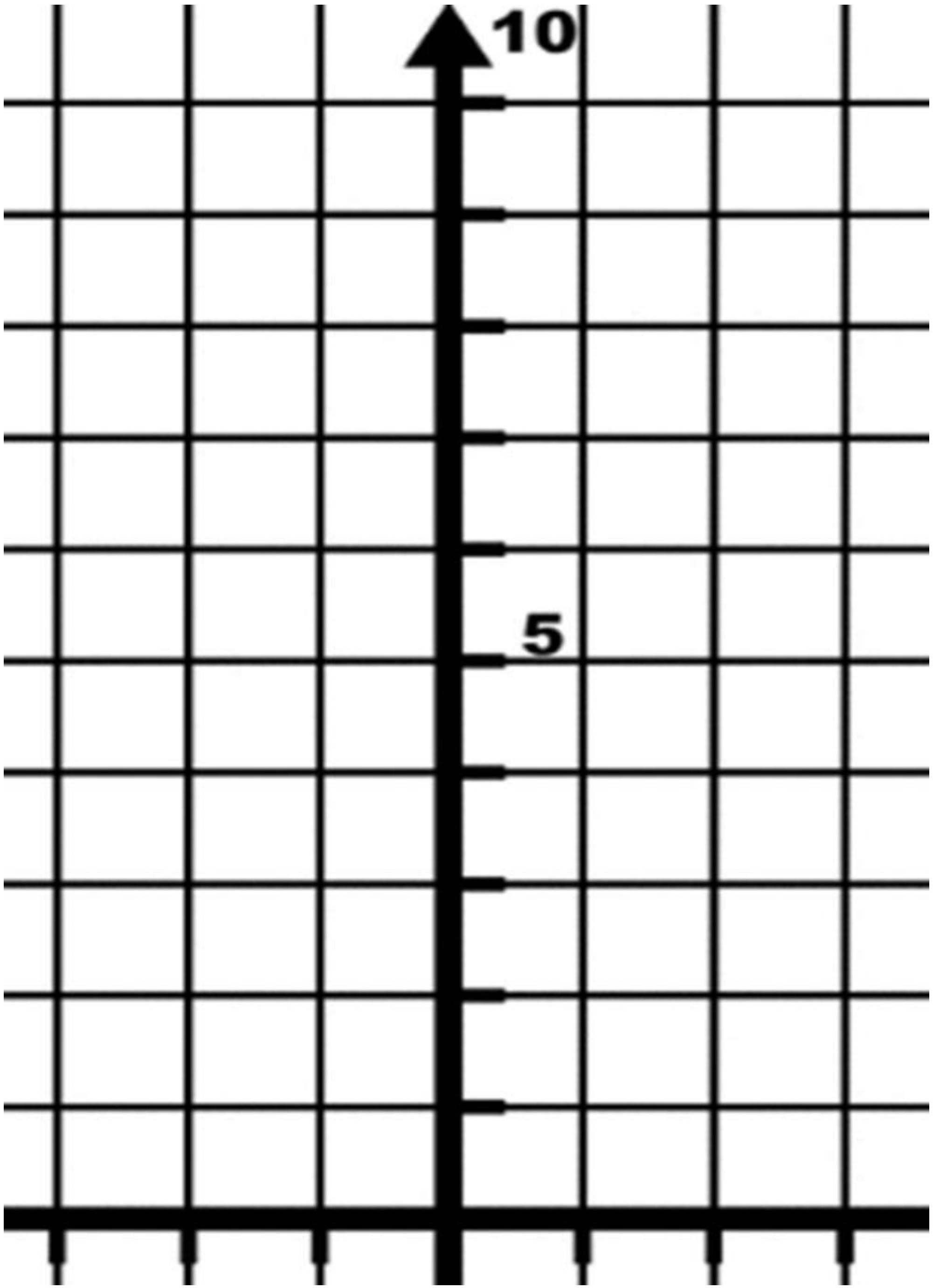
The rate of change of x is 2 units/second. What is the rate of change of Y?

$$\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = ?$$

$$\frac{d}{dt}[y = x^2 + 1]$$

$$\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dy}{dt} = 4x\frac{units}{second}$$



$$y = x^2 + 1$$

$$\frac{dy}{dt} = 4x \, \frac{units}{second}$$

What if we are in the first quadrant?

What is $\frac{dy}{dt}$**?**

What if we are in the second quadrant?

What is $\frac{dy}{dt}$?

How about if we are at (0,0)?

Does this make sense??

Some Considerations…

- If something is increasing with time, the derivative will be __**positive**__.
- If something is decreasing with time, the derivative will be__**negative**__ .
- If something is constant over time, then the derivative is ____**0**____.

Area and Radius of a Circle

The radius of a circle is increasing at a rate of 5 cm/min. At what rate is the area growing at the moment the radius is 7 cm?

- The radius of a circle is 7 cm? At this moment, at what rate is area changing with respect to radius?

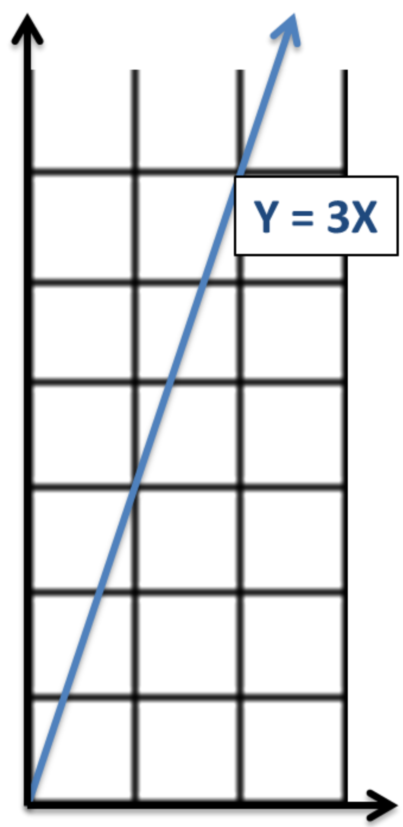
- What is the relationship between this question and the previous one?

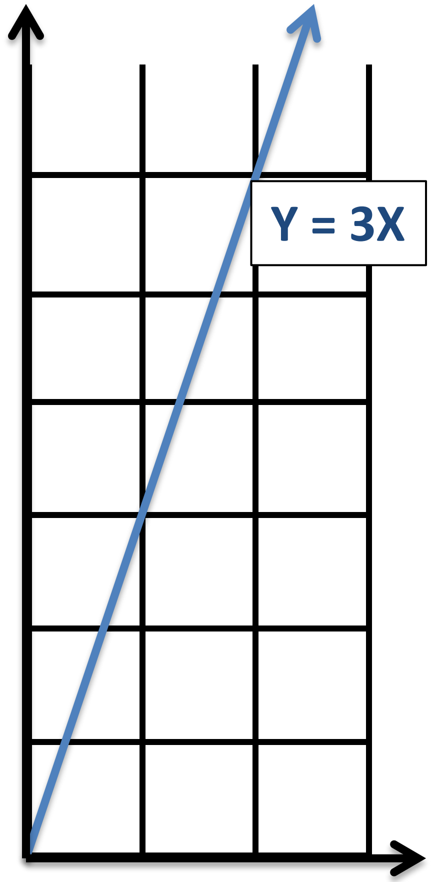
Homework Day 1
(1-11 odd, 10, 13, 15, 16, 18)

Distance from Origin.

A particle is traveling along the line Y = 3X at a horizontal rate of 2 units/second ($\frac{dx}{dt} = 2$).

Find the rate of change of the distance between the origin and a moving point at (2, 6).

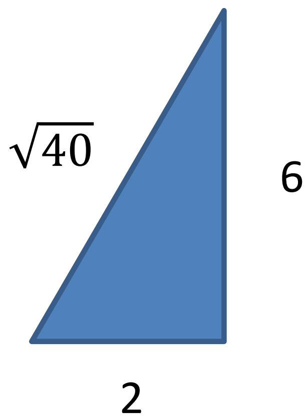




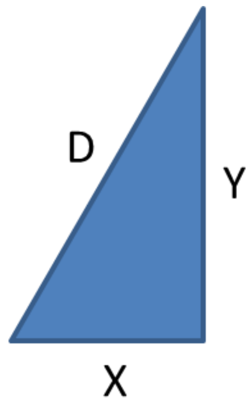
$$\frac{dx}{dt} = 2 \qquad \frac{dy}{dt} = 6$$

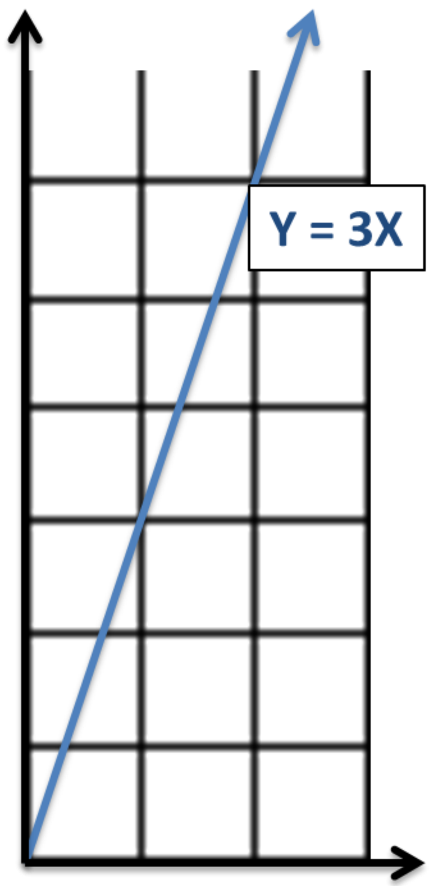
So, how fast is it traveling away from the origin?

Dynamic Model



If it is traveling at a rate of 2 units/sec horizontally and 6 units/sec vertically, it must be traveling at a rate of $\sqrt{40}$ units/sec away from the origin.





Solving this problem Analytically:

We need a formula that relates D, X and Y.

$$D = \sqrt{X^2 + Y^2}$$

$$D(t) = [(X(t))^2 + (Y(t))^2]^{\frac{1}{2}}$$

$$\frac{dD}{dt} = \frac{1}{2}[X^2 + Y^2]^{-\frac{1}{2}} \cdot \left[2X\frac{dx}{dt} + 2Y\frac{dy}{dt}\right]$$

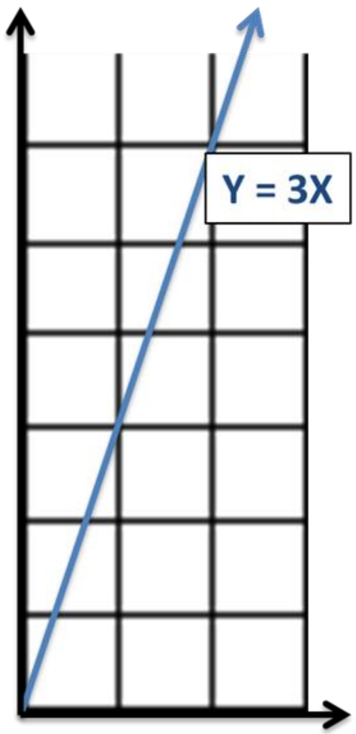
$$\frac{dD}{dt} = \frac{X\frac{dx}{dt} + Y\frac{dy}{dt}}{\sqrt{X^2 + Y^2}}$$

Now , you should plug in to solve for $\frac{dD}{dt}$

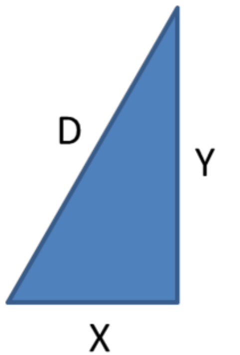
$$\frac{dD}{dt} = \frac{X\frac{dx}{dt} + Y\frac{dy}{dt}}{\sqrt{X^2 + Y^2}}$$

What is the rate of travel from origin at (1,3)?

Y = 3X and $\frac{dx}{dt} = 2$



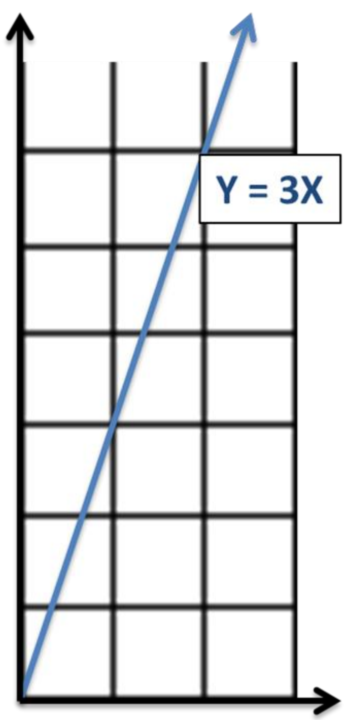
Method 2: D = $\sqrt{x^2 + y^2}$



We know that $y = 3x$ and $\frac{dx}{dt} = 2$

D = $\sqrt{x^2 + (3x)^2}$

= $\sqrt{x^2 + 9x^2}$ = $\sqrt{10x^2}$



$$\frac{dD}{dt} =$$

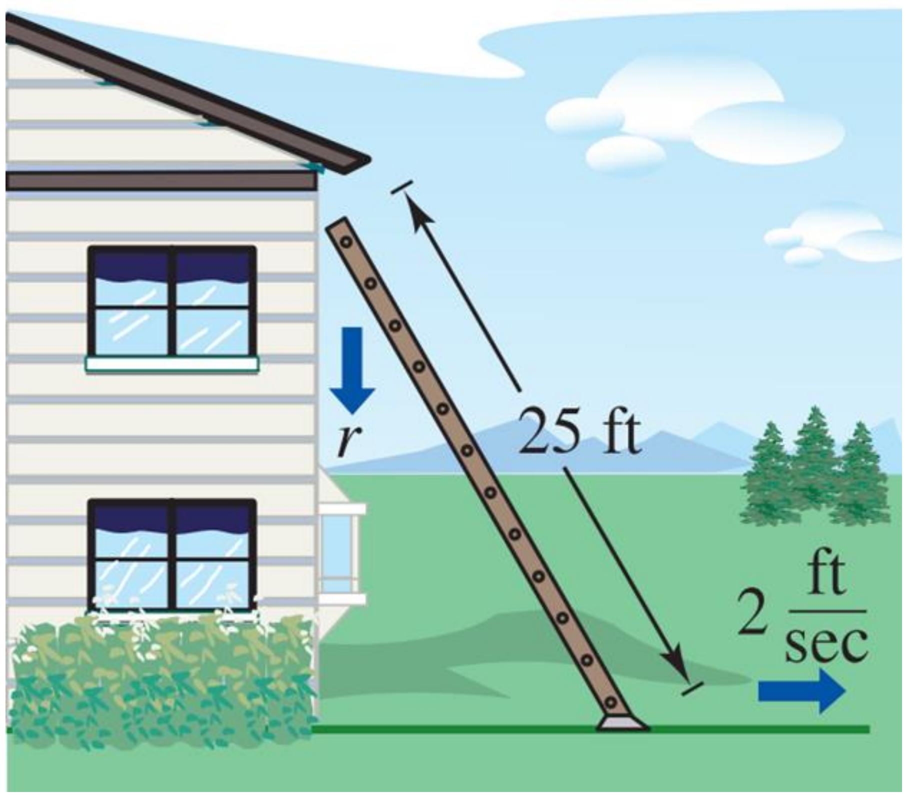
$$\frac{dD}{dt} =$$

If you can ever eliminate a variable it will make your life much easier. ☺

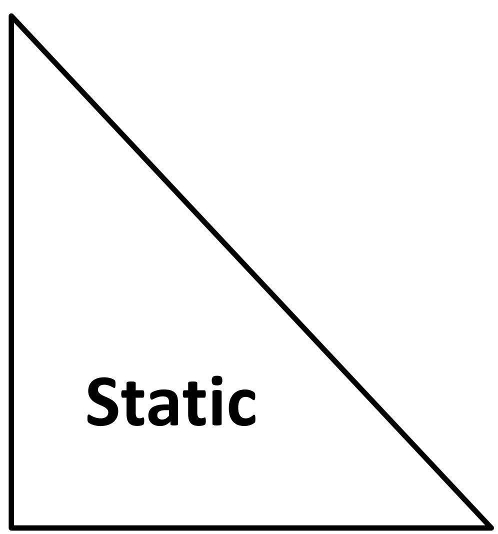
A spherical balloon is inflated at a rate of 500 cubic centimeters per second. How fast is the radius of the balloon increasing at the instant the radius is 12 cm? $v = \frac{4}{3}\pi r^3$

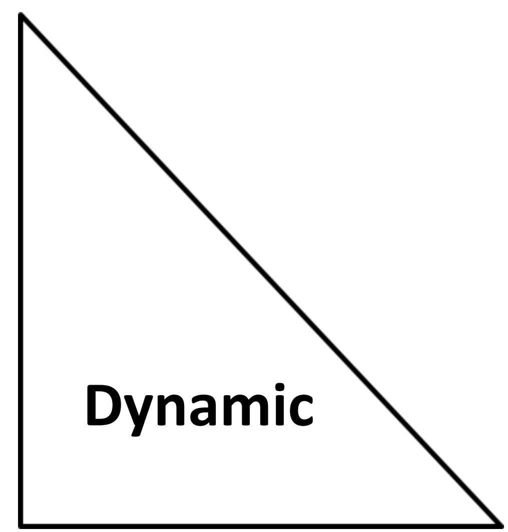
All sides of a cube are expanding at a rate of 5 cm/sec. How fast is the volume changing when each edge is 2 cm? The volume of a cube is given as $V = s^3$.

A 10 foot ladder is leaning against a vertical wall. The ladder slides away from the wall in such a way that distance between the foot of the ladder and the wall is increasing at a constant rate of 0.5 ft/sec. At what rate is top of ladder sliding down the wall when the ladder is 6 ft from the wall?



A 10 foot ladder is leaned against a vertical wall. The ladder slides away from the wall in such a way that distance between the foot of the ladder and the wall is increasing at a constant rate of 0.5 ft/sec. At what rate is top of ladder sliding down the wall when the ladder is 6 ft from the wall?





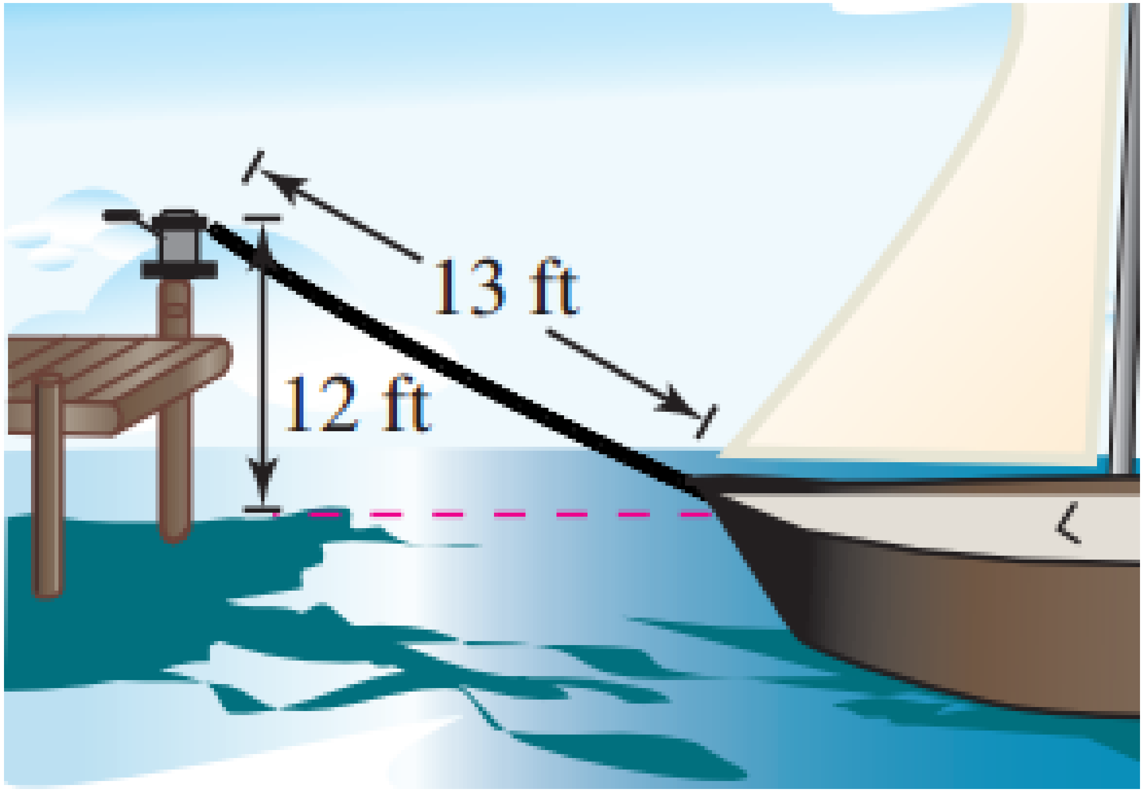
What is the formula that relates the unknown quantities?

$$X^2 + Y^2 = L^2$$

Related Rates Word Problems.

1. Draw a picture to represent the problem.
   (sometimes draw 2, one static and one dynamic)
2. Determine a formula that relates the quantities in question.
3. Minimize the number of variables by:
   A. Substituting known relations
   B. Substituting CONSTANTS
4. Differentiate with respect to Time.
5. Solve for the unknown value.

A boat is being pulled towards a dock by a pulley that is located 5 ft above the bow of the boat. The line is being reeled in at a constant rate of 4 ft/sec. When there is 13 feet of line between the pulley and the boat, at what rate is the boat moving over the surface of the water?



Homework Day 2

Related Rates Worksheet

(14, 19, 20, 22, 30, 31)