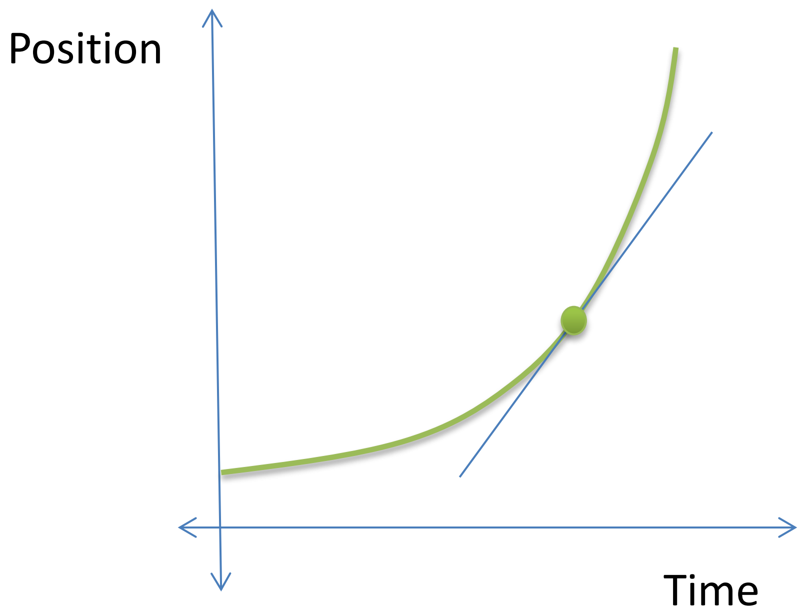
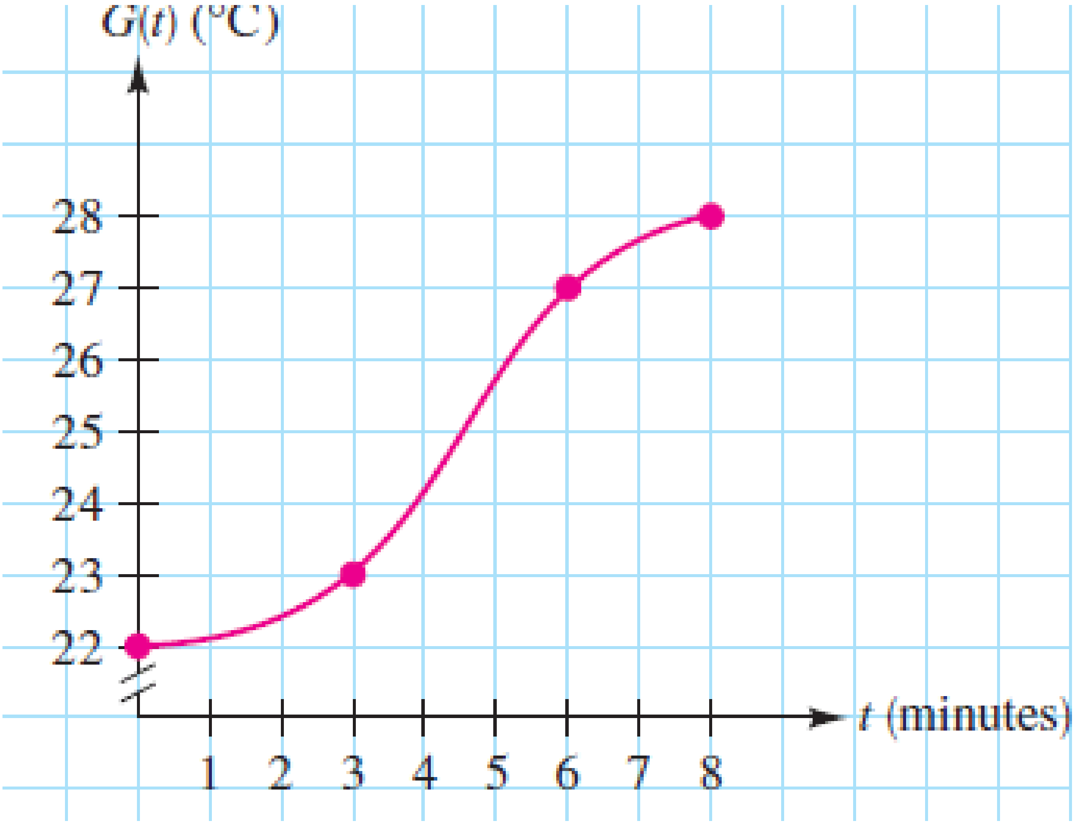
VELOCITY

If you have a graph of position and you take the derivative, what would the derivative represent?



Average rate of Change



What is the average rate of change of temperature over the first 6 minutes (from $t = 0$ to $t = 6$) ?

In words, how can you write the average rate of change of a quantity?

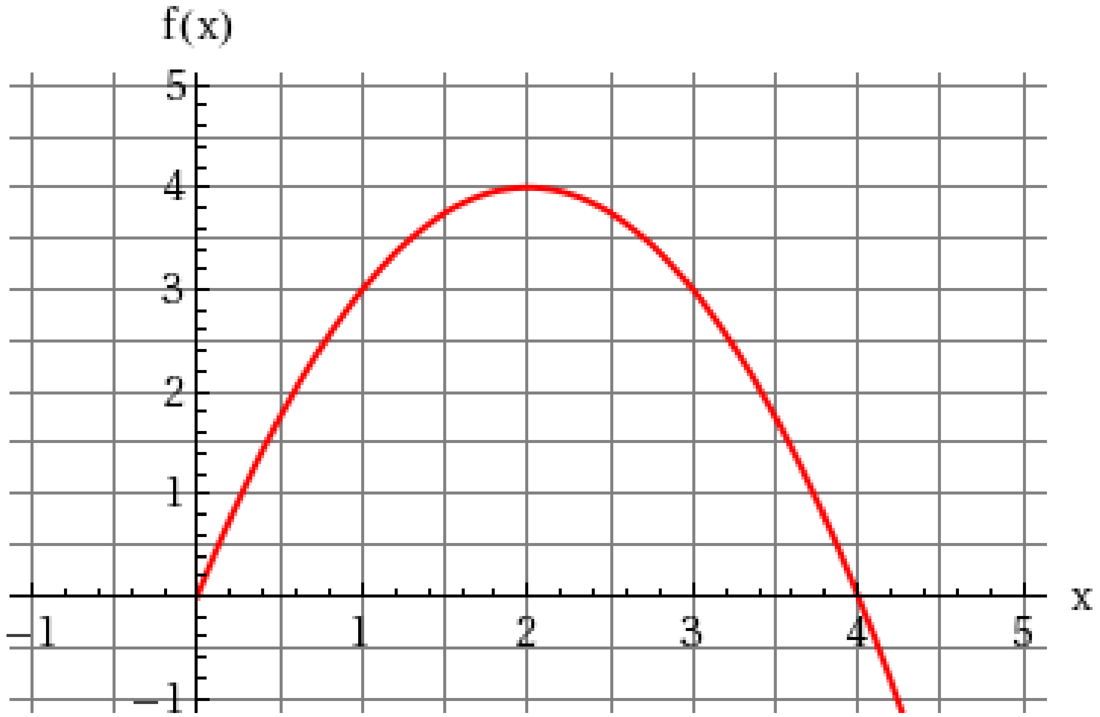
$$\frac{Total\ Change\ of\ Quantity}{Total\ Time}$$

Graphically, the Average Rate of Change can be thought of as:

$$\frac{Total\ Vertical\ Change}{Total\ Horizontal\ Change} = \frac{\Delta y}{\Delta x}$$

Average Rate of Change over the interval $[a, b] = \frac{f(b)-f(a)}{b-a}$

(The slope over the interval)



What is the **average rate of change** over the following intervals? What units would you use for the average rate of change?

$[0, 2]$ $[0, 3]$ $[1, 3]$

“Average of all the tangential slopes over the interval”

Imagine you drive to Guangzhou from Shenzhen. It takes you two hours and the distance is 150 km.

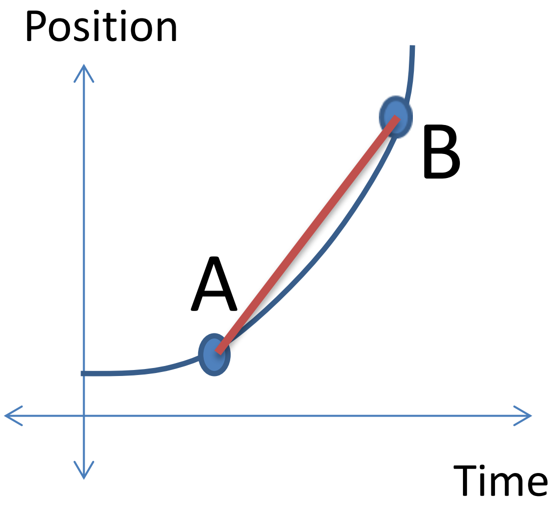
What is the average velocity?

In words, what is the average velocity?

$$= \frac{total\ distance}{total\ time}$$

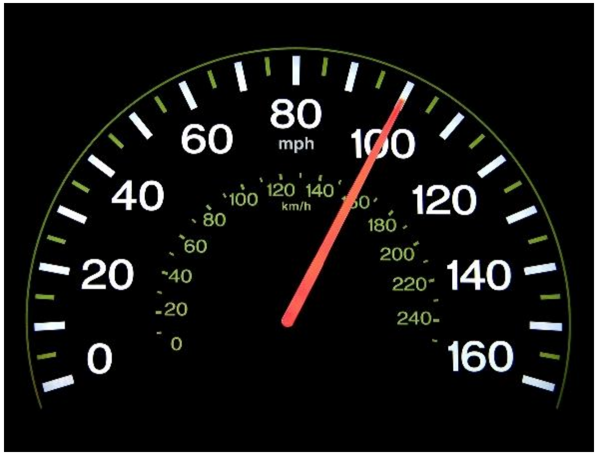
Average Velocity

- Average Velocity is the TOTAL DISTANCE divided by the TOTAL TIME.
- The Average Velocity is the SLOPE of the POSITION FUNCTION from A to B.

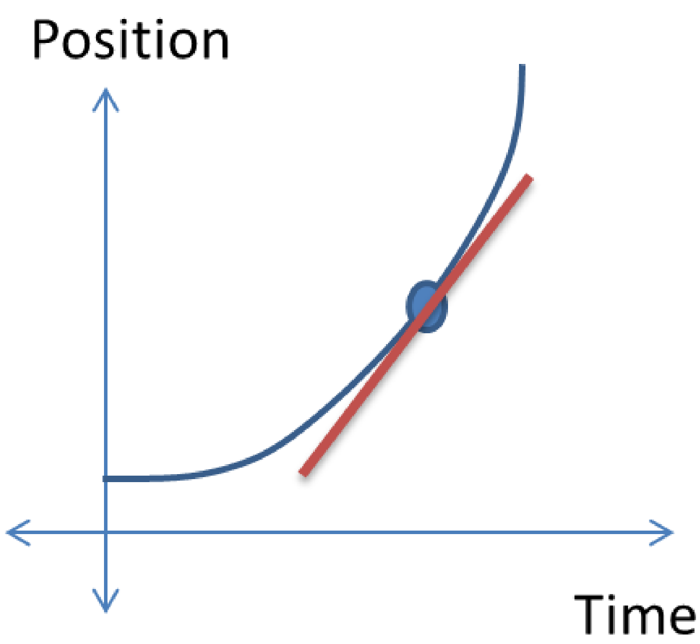


Instantaneous Velocity

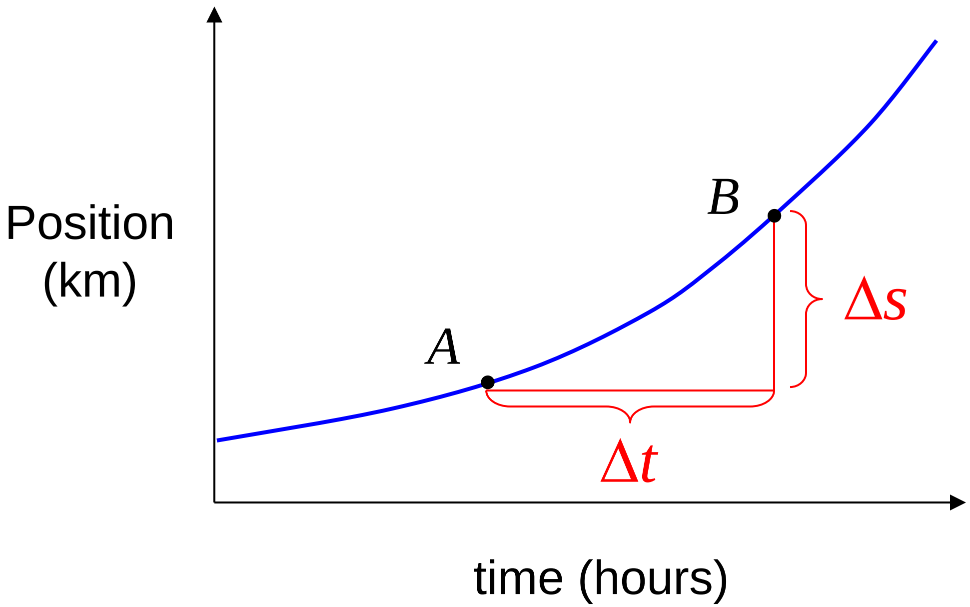
Instantaneous velocity is given to us by the speedometer on a car.



The instantaneous velocity is found by taking the derivative of the position function.



Velocity is the derivative of Position.



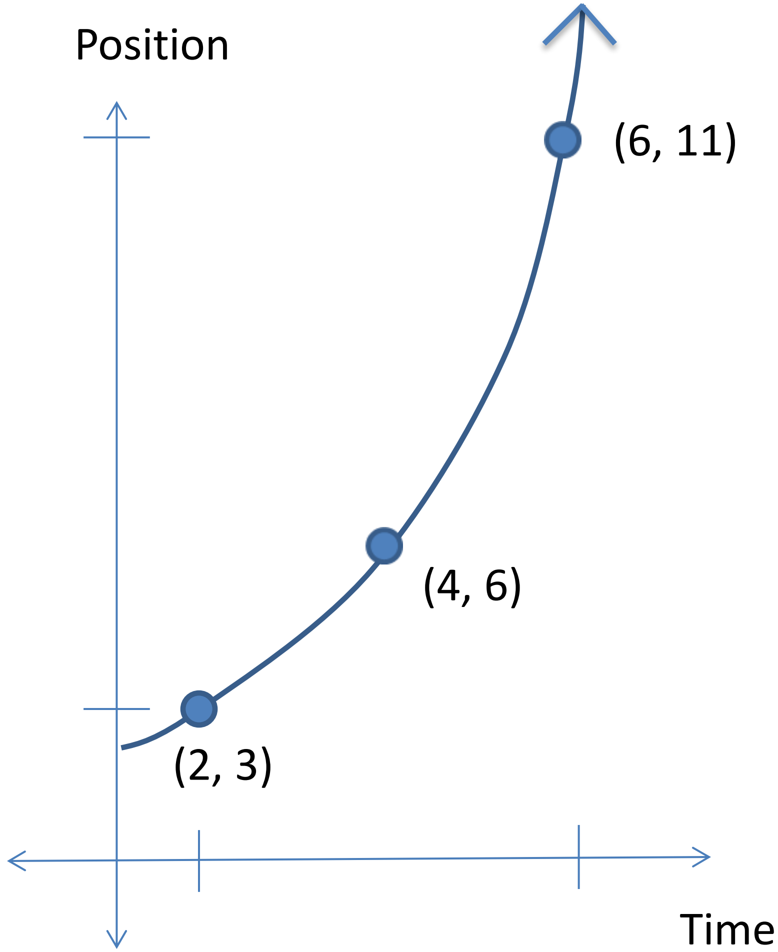
What is the Average velocity on interval $[a, b]$?

$$\frac{\text{change in position}}{\text{change in time}} = \frac{\Delta s}{\Delta t}$$

If we let point A be $(t, f(t))$

$$V_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

$$V(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$



Slope = Velocity

∴

Derivative = Instantaneous Velocity

The position of a function is given by:

$$y = \frac{1}{4}t^2 + 2$$

What is the average velocity from [2,6]?

What is the instantaneous velocity at:

$t = 2$

$t = 4$

$t = 6$

Velocity is the derivative of position.

Velocity can be either positive or negative.

If moving to LEFT or DOWN, velocity NEGATIVE
If moving to RIGHT or UP, velocity POSITIVE

The Instantaneous speed on the other hand is always positive, it is the absolute value of Instantaneous Velocity

$$S(t) = |v(t)|$$

***Keep in mind we are only dealing with one-dimensional motion.**

Higher Order Derivatives

Higher Order Derivatives are where you repeatedly take a derivative.

$$f(x) = 5x^5 - 3x^3 + 4x$$
$$f'(x) = 25x^4 - 9x^2 + 4$$
$$f''(x) = 100x^3 - 18x$$
$$f'''(x) = 300x^2 - 18$$

Acceleration

What is the relationship, graphically, of Acceleration to Velocity and Position?

Acceleration

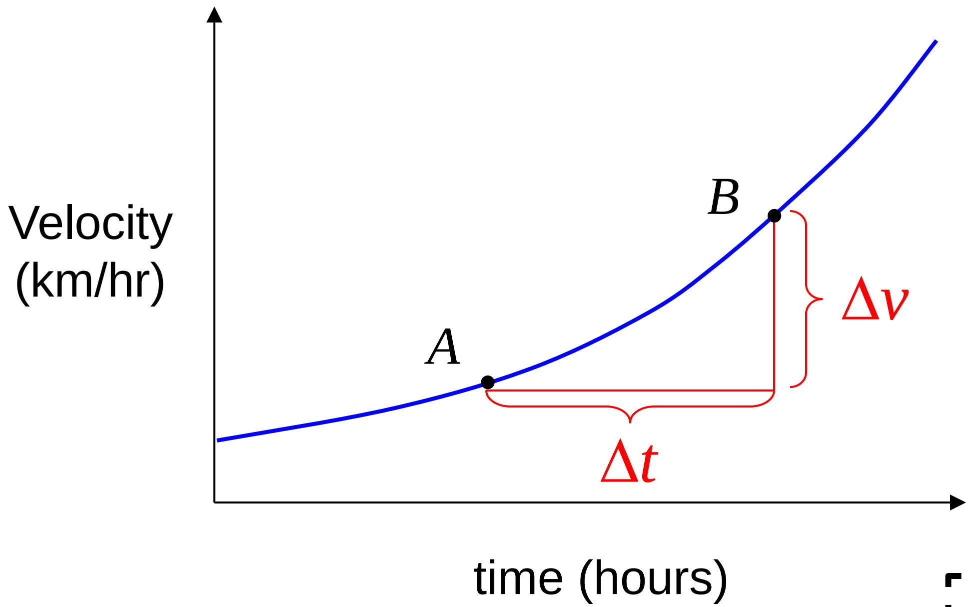
-The second derivative of position.

-The (first) derivative of velocity.

Acceleration is the derivative of velocity with respect to time. If a body's velocity at time t is given by $v(t) = \frac{ds}{dt}$ then the acceleration is:

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Acceleration is the derivative of Velocity.



What is the Average Acceleration on interval $[a, b]$?

$$\frac{\text{change in velocity}}{\text{change in time}} = \frac{\Delta v}{\Delta t}$$

If we let point A be $(t, v(t))$

$$\boldsymbol{A_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(t+\Delta t) - v(t)}{\Delta t}}$$

$$\boldsymbol{A(t) = \frac{dv}{dt} = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t}}$$

Position, Velocity and Acceleration

Position:

$$S(t) = 5t^3 + 4t^2 - 1$$

Velocity:

$$V(t) = 15t^2 + 8t$$

Acceleration:

$$A(t) = 30t + 8$$

An object falls from a 100 meter mountain

Position

$$s(t) = -4.9t^2 + 100 \text{ (meters)}$$

Velocity

$$v(t) = -9.8t \text{ (m/s)}$$

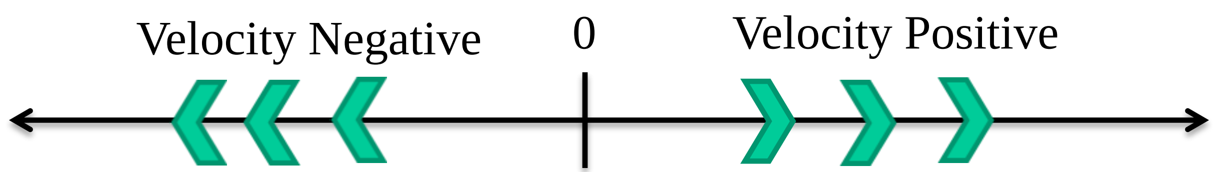
Acceleration

$$a(t) = -9.8 \text{ (m/}s^2\text{)}$$

(acceleration due to gravity)

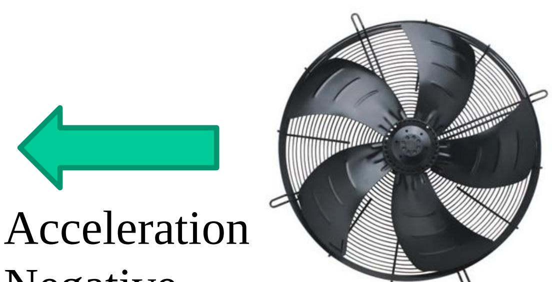
Velocity or Acceleration to left or down is NEGATIVE

Velocity or Acceleration to right or up is POSITIVE



Think of acceleration as a force

Acceleration Positive



Acceleration Negative

If Velocity and Acceleration have same sign (either both positive or both negative) is the object speeding up or slowing down?

Speeding Up!!!

If Velocity and Acceleration have opposite signs (one positive and the other negative) is the object speeding up or slowing down?

Slowing Down!!!

Group Activity

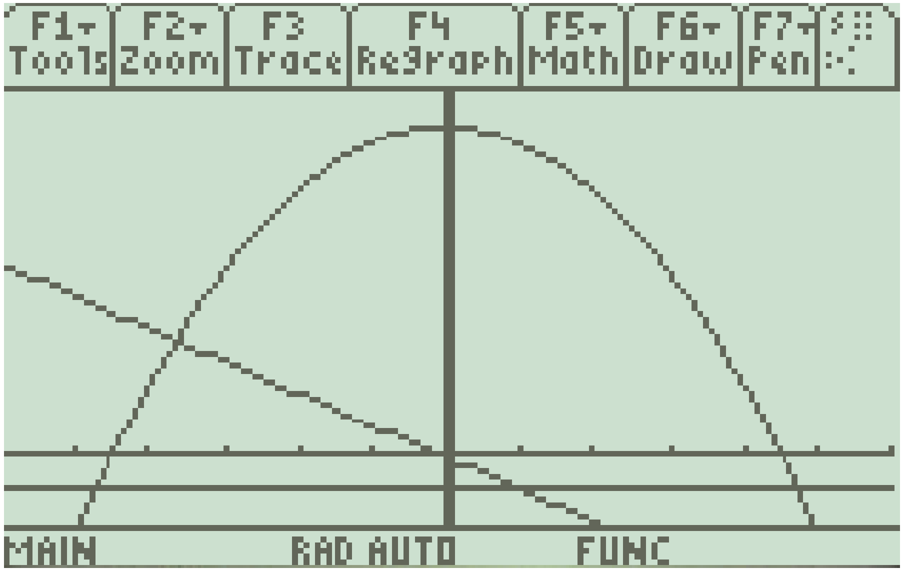
Section 3.4:

Velocity and Other Rates of Change

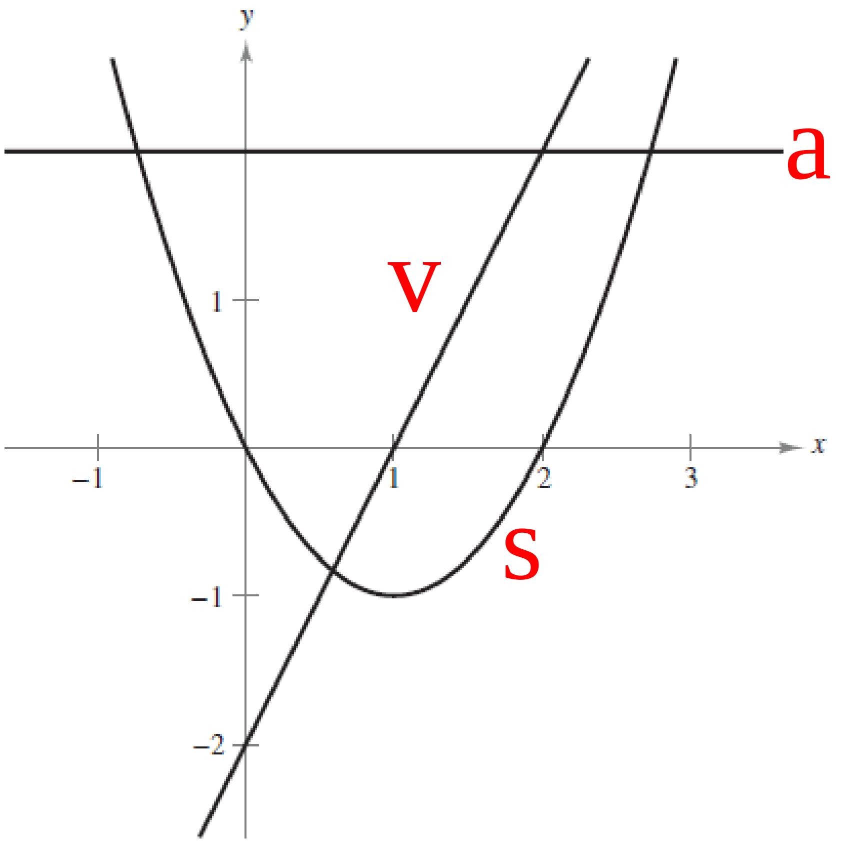
P. 138 (orange book)(31 and 32)

Graph these three on the same graph. Do the graphs make sense?

Position:	$s(t) = -4.9t^2 + 100$ (meters)
Velocity:	$v(t) = -9.8t$ (m/s)
Acceleration:	$a(t) = -9.8$ (m/s^2)



Label each graph as Position (s), Velocity (v) or Acceleration (a)



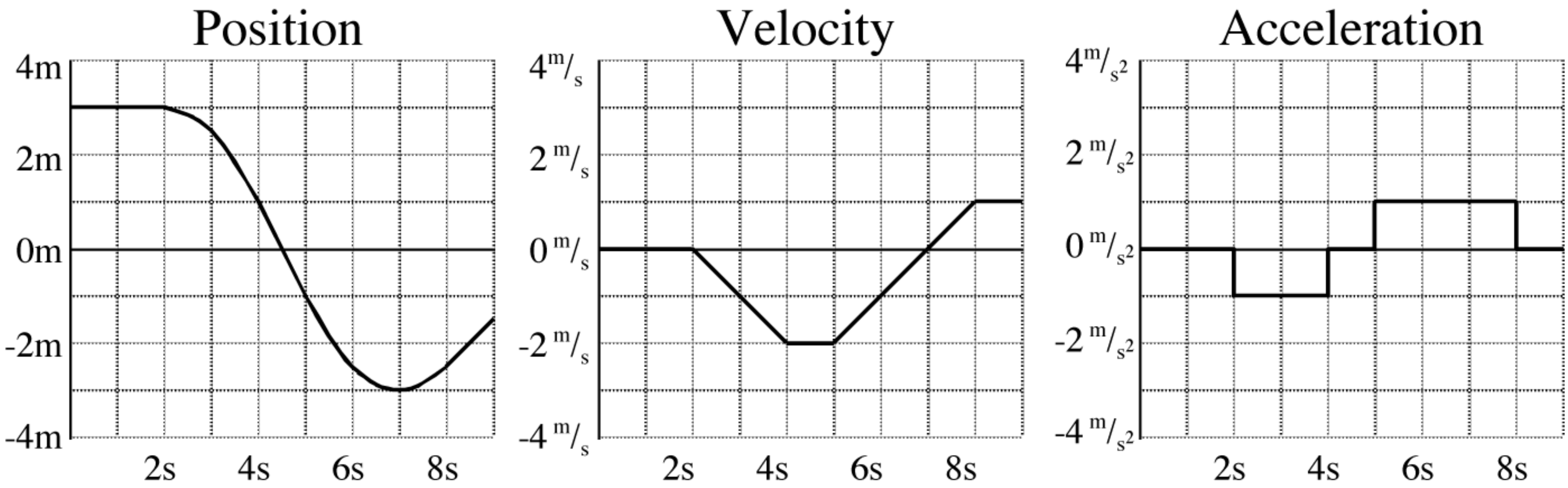
The derivative of velocity is acceleration.

The antiderivative of acceleration is velocity.

Acceleration is Slope of Velocity

Since Acceleration is the Derivative of Velocity,

The **slope** of the **velocity** graph gives **acceleration**.



Homework

Orange Book

P. 136 (8, 9, 11, 13, 15, 19)

Homework:

Sec. 2.2

P. 117 (89-97 odd) (Vel./Acc)

Sec. 2.3

P. 129 (95, 97, 99, 109-115 odd, 118, 125, 127, 129-135)

(2nd Deriv. and Velo/Acc.)

Derivatives and Economics

P. 133-134 of book

Orange Book
P. 136 (27, 28)